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ABSTRACT

This document outlines the content standards for mathematics in the state of North Carolina. It is organized into four sets of grade levels: K-2; 3-5; 6-8; and high school courses. This guide establishes competency goals and objectives for the teaching and learning of mathematics in North Carolina. Outlines of the state position on such topics as technology, student learning, and professional development are also included. The standards themselves list competency objectives for students at various pre-high school grade levels under four headings: (1) Number Sense, Numeration and Numerical Operations; (2) Spatial Sense, Measurement, and Geometry; (3) Patterns, Relationships, and Functions; and (4) Data, Probability, and Statistics. High school standards are addressed on a course-by-course basis for Algebra I, Technical Mathematics, Integrated Mathematics, and others. (MM)

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ED 457 026

# Revised North Carolina Mathematics Standard Course of Study

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The State Board of Education approved revisions to the North Carolina Mathematics Standard Course of Study at the May 1998 monthly meeting.

The revised Mathematics Standard Course of Study is to be used for instructional planning and textbook selection during the 1998-99 school year.

The full implementation of the curriculum revisions takes place during the 1999-2000 school year.

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# Mathematics

# K-12

# Standard Course of Study and Grade Level Competencies

# FOREWORD

North Carolina has had a ***Standard Course of Study*** since 1898. Since that time, the curriculum has been revised periodically to reflect the changing needs of students and society. The most recent revision of the state mathematics curriculum occurred in 1989 (K-8) and 1992 (9-12). Those curricula reflected the shift in the knowledge, skills, and attitudes needed by business, industry, and society to function in an information-driven world. The current revisions continue to build upon those efforts. Based upon the work of the North Carolina Mathematics Framework Committees, the ***North Carolina Mathematics Standard Course of Study*** Committees constructed a curriculum focused on giving students the opportunity to:

- acquire the mathematical literacy necessary to function in an information age,
- cultivate the understanding and application of mathematical skills and concepts necessary to thrive in an ever-changing technological world,
- develop the essential elements of problem solving, communication, reasoning, and connections within their study of mathematics, and
- understand the major ideas of mathematics.

The ***North Carolina Mathematics Standard Course of Study*** clearly defines a curriculum supporting the ABC's school reform effort as well as the North Carolina Testing Program. These revisions maintain a forward focus, looking at what students will need to know and be able to do to be successful and contributing citizens in our state and nation in the years ahead.

# ACKNOWLEDGMENTS

The Department of Public Instruction gratefully acknowledges the cooperation and assistance received from individuals and groups throughout the State in this current revision process. Without such cooperation, the revisions and printing of the ***North Carolina Standard Course of Study*** would not have been possible.

We wish to express a special thanks to:

- the Office of Instructional Services for providing the leadership and vision that guided the development of this document,
- the many local educators, parents, and business people who participated in the current revision process by serving on curriculum committees and reacting to draft documents,
- faculty from the institutions of higher education who advised the staff and assisted in the revision of the curriculum, and
- the Department of Public Instruction staff who carried the primary responsibility for revising and editing the curriculum.

The current revision process involved on some level the entire mathematics education community, and its end product is a North Carolina curriculum of which North Carolina can be proud. We will regularly revise and improve the curriculum in order to meet the needs of the students of North Carolina.

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# PREFACE

**Intent**

The intent of the *North Carolina Mathematics Standard Course of Study* is to establish competency goals and objectives for the teaching and learning of mathematics in North Carolina. This document is only the first in a series; additional documents will provide more detailed recommendations and support for implementation.

The primary goal of mathematics education in North Carolina is to ensure that all students develop *mathematical power*. "This term denotes an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems." (p.5, *Curriculum and Evaluation Standards for School Mathematics*, NCTM)

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**Revisions**

The *North Carolina Mathematics Standard Course of Study* was last revised in 1989 (K-8) and 1992 (9-12). Reforms and revisions in mathematics education programs to foster the development of mathematical power are necessary because:

- The mathematics required for competence in the work place continues to change and increase;
- The population that needs mathematical competence has significantly expanded; and
- Research has greatly advanced our knowledge about the teaching and learning of mathematics.

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**Changing Vision of Mathematics**

Students in North Carolina have demonstrated progress in mathematics learning in recent years. To build on this improvement, it is imperative that broad-based implementation of mathematics reforms continue so that all students in our state can realize their potential.

Because mathematics and particularly the methods and tools for doing mathematics are so different today from a generation ago, the nature of mathematics is changing. Traditional school mathematics is no longer a sufficient preparation for today's students. A revised perception of teaching is required in order for mathematical power to be achieved.

One important change is the rapid expansion of the body of mathematical knowledge; furthermore, mathematical applications have more impact on citizens. For example, the quality of our lives is enhanced by the mathematical modeling that is used to predict storms, manage our natural resources, and handle vast amounts of information.

Some of the complex mathematical procedures that today's adults learned when they were in school, such as computing square roots and doing tedious calculations, are no longer as important as they once were. Because of

electronic computing and information storage devices, adults are no longer required to be able to carry out complex paper and pencil computations or maintain extensive records on paper.

Some mathematics is more accessible to today's students due to new knowledge and technology. Software on graphing calculators and computers allows students to investigate aspects of geometry and algebraic functions that are impossible to investigate with paper and pencil alone.

For these reasons students need a strong understanding of our number system, and the ability to estimate and compute mentally. Fundamental to these skills are knowledge of number facts, the basic computational processes and the appropriate use of each operation. Together with an emphasis on using mathematics to solve problems, the mathematics curriculum will provide students with depth of understanding and skill in applying the content.

**Technology**

Technology has changed both the mathematics that is important for solving real-world problems and the ways that mathematics is used to solve those problems. North Carolina workers who assemble furniture, for instance, are now expected to have problem-solving skills with which they monitor production lines and make decisions which may halt production, whereas in the past, such workers would have been expected only to follow repetitive procedures without making any decisions. Thus, problem solving has become important for all students and needs to be reflected both in content of school mathematics and in the ways that mathematics is taught and assessed.

**Relevant Mathematics**

A second important change is that the demands of contemporary society in general and the work place in particular increasingly require higher levels of sophistication in mathematical thinking. During the industrial era, good jobs were available for large numbers of relatively low-skilled workers. That day is gone. Mathematical power for all is becoming more and more essential for a healthy economic future. Therefore, all students should continue to study significant mathematics throughout their public school experience, including the development of good number and spatial sense, knowledge of patterns and functions, and the ability to gather, represent, analyze, and interpret data.

**Student Learning**

A third change is that research on how students learn mathematics has begun to reveal critical details about how students internalize important mathematics concepts. There is compelling evidence that students are far more capable mathematically than we ever imagined. As a result, we can now plan instruction in ways that are for more likely to help them develop mathematical power.

One way is the use of mathematical modeling, a key technique used to build understanding of abstract ideas. Teachers need to expose students to physical representations which help develop understanding of abstract concepts. Early years should include work with manipulatives to help form a sense of number; geometric shapes and patterns facilitate the development of spatial reasoning. In later studies, students will generate algebraic expressions, another form of modeling, which represent physical, social, or natural phenomena and help them make predictions.

One of the challenges facing education today is the development of effectice mechanisms for informing teachers about this research so that they can transform the learning environment in their classrooms. Research shows that students develop mathematical competence and power by engaging in solving meaningful problems. Beginning in the earliest grade levels, students should be challenged to use their own knowledge and experience, working alone, in pairs, and in small and large groups, to solve problematic tasks. They should be expected to communicate their thinking with pictures, numbers and words. Teachers should encourage students to question one another when an explanation doesn't make sense to them. This problem-centered approach to learning mathematics will enable students to take greater responsibility for their own learning, to develop essential communication and decision-making skills, and to understand the fundamental concepts of mathematics, all of which will be critically important to them.

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**Program Review**

These changes require a reevaluation of all aspects of mathematics education. There is an urgent need to reexamine:

- The roles of teachers and students in classrooms;
- The content of school mathematics;
- Assessment practices;
- The preparation and professional development of teachers and
- The level of support for mathematics education from all parts of society.

## Mathematics for all North Carolina students

### Goals

The vision and philosophy described throughout this document are based on our goals in mathematics education for North Carolina students.

The six goals are for *all* students to develop:

- Strong mathematical problem solving and reasoning abilities;
- A firm grounding in essential mathematical concepts and skills, including computation and estimation;
- Connections within mathematics and with other disciplines;
- The ability to use appropriate tools including technology to solve mathematical problems;
- The ability to communicate their understanding of mathematics effectively; and
- Positive attitudes and beliefs about mathematics.

These goals for our students in mathematics are the foundation for the rest of the document and guide the development of the critical areas in mathematics education.

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### Critical Aspects of Mathematics Education

Five components have been identified as critical for achieving the goals for our students and as making a significant impact on the quality of mathematics education. A summary of each of these is given here. They are:

- Teaching and Learning
- Content
- Assessment
- Preparation and Professional Development of Teachers
- Roles and Responsibilities

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### Teaching and Learning

Teachers are the key to changing the learning environment in North Carolina's classrooms. They plan classroom experiences and create a supportive environment for learning to take place. A teacher plays many roles in today's classrooms as the guide, the coach, the facilitator, and the instigator of mathematical explorations. Through their classroom practices, teachers promote students' mathematical reasoning, challenge them with rich problems through which they learn to value mathematics, and provide them with a strong foundation for further study. Most of all, teachers encourage and support their students' development of mathematical power.

The heart of mathematics is reasoning, which together with knowledge of appropriate mathematical content forms the basis of mathematical power. The goal of mathematics education in North Carolina is to enable all students to develop mathematical power and confidence in their ability to do mathematics. A curriculum that focuses on significant mathematical ideas, instead of isolated topics, encompasses both concepts and skills through rich explorations, problems, and applications that enable students to develop a genuine understanding of the big ideas of mathematics.

## Assessment

Assessment is a process of planning, gathering evidence, interpreting evidence, and making decisions. Mathematics assessment is directly related to instruction and student thinking. Four purposes of assessment are monitoring student progress, making instructional decisions, evaluating student achievement, and evaluating a program.

Assessment has taken on a broader meaning. Beyond grading students, assessment should probe beneath right answers to discover how students think and how instruction can be improved. In this view of assessment, expected outcomes are set and the time necessary for each student to achieve the intended outcomes varies. Every student is challenged to meet a higher standard.

Effective assessment fosters the development of mathematical power. Students must be given opportunities to acquire and demonstrate understanding and depth of knowledge. The criteria for judging mathematical accomplishment must be made public and communicated clearly to students, parents, and other appropriate parties.

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## Preparation and Professional Development of Mathematics Teachers

Teaching in a manner that cultivates mathematical power for all students is a complex and demanding process that requires intensive lifelong learning. Teachers must have not only extensive knowledge of mathematics, but also deep understanding of how students learn mathematics. Appropriate content and pedagogical preparation enables teachers to design lessons and implement curriculum using suitable strategies and resources in an environment where all students have an opportunity to succeed. Teachers are role models for their students, as learners and problem solvers who value and enjoy mathematics. Programs for teachers at all stages in their career must provide them with the tools to implement the goals previously stated on page 4.

The professional development of a mathematics teacher is an ongoing process. This development occurs in three distinct phases: preservice (undergraduate or teacher preparation); induction (the first 3 to 5 years of teaching); and in-service (continued professional growth over the span of the teacher's career). Communication and articulation efforts to link these phases are necessary for a continuum of professional growth. Although teachers need to take a major responsibility for their own professional development, it also requires leadership, resources, financial support, and advocacy at the state, district, school, classroom, and university levels.

## Roles and Responsibilities

Many different constituencies in North Carolina must work together to establish a mathematics education program that enables all students in the state to develop their full potential as powerful and creative thinkers and problem solvers.

Support for mathematics education requires that:

- Legislators and other financial partners provide funding that increases the likelihood of student achievement;
- School boards and administrators enact policies which enable teachers to provide quality instruction;
- Colleges and universities help teachers develop content knowledge and expertise in teaching;
- Parents and other citizens become partners with schools to value and nurture student efforts; and
- Educators and students work cooperatively to establish and reach high educational goals.

The education of students is our shared responsibility. All citizens of North Carolina must assume this responsibility and become active advocates for improved mathematics education.

# Mathematics Curriculum

**Organization**

The competency goals and objectives of the mathematics curriculum are organized into four strands: Number Sense, Numeration, and Numerical Operations; Spatial Sense, Measurement, and Geometry; Patterns, Relationships, and Functions; Statistics, Probability, and Discrete Mathematics. These strands are not meant to be a sequential guide for instruction but rather an organization of similar objectives under a common topic.

The mathematics program is designed in grade spans which parallel the developmental stages of students: grades K-2, grades 3-5, grades 6-8, and grades 9-12. The elementary program focuses on students actively engaged in the development of mathematical understanding by using manipulatives, working independently and cooperatively to solve problems, and conducting investigations and recording findings. Middle grade students expand their skills to compute with all real numbers and are challenged to apply their prior knowledge and experience in new and more difficult situations. The basic high school mathematics program includes courses from Introductory Mathematics through Advanced Advanced Placement Calculus. Additional elective courses are intended to offer opportunities which address the needs of individual schools.

| **K - 2, 3 - 5, 6 - 8**<br>**Mathematics** |
|---|
| **High School Courses**<br>•*Introductory Mathematics* •*Algebra I* •*Algebra II*<br>•*Geometry* •*Technical Mathematics 1* •*Technical Mathematics 2*<br>•*Advanced Mathematics* •*Advanced Placement Calculus* |
| **Additional Electives**<br>•*Integrated Mathematics 1, 2, 3*<br>•*Discrete Mathematics*<br>•*Advanced Placement Statistics* |

**Revisions**

Notable differences for 1998 revisions to the *North Carolina Mathematics Standard Course of Study* include the following:

- Seven strands collapsed into four strands.
- Repetition of concepts and skills across grades eliminated.
- Introduction of an Integrated Mathematics sequence at the secondary level.
- New high school elective courses: Advanced Placement Statistics, Technical Mathematics II, Discrete Mathematics.

# K-8
# MATHEMATICS
# COMPETENCY GOALS
# AND OBJECTIVES

PUBLIC SCHOOLS OF NORTH CAROLINA DEPARTMENT OF PUBLIC INSTRUCTION

Revised 1998

# Early Grades K-2

**Number Sense, Numeration, and Numerical Operations**

During the early years in a curriculum designed to develop number sense, students read, write, and count using whole numbers. They represent whole numbers using concrete, pictorial, and symbolic representations. They recognize different representations for whole numbers and explain why those representations are the same. They compare and order whole numbers and use a variety of strategies to estimate quantities.

The basic understanding of place value forms the foundation for subsequent work with number. Students group objects recognizing that digits have different values depending upon their placement or position in numerals.

Students use many tools and strategies to model solutions for a wide variety of mathematical story problems involving addition, subtraction, multiplication, and division, even though they might not be able to write symbolic representations in all of these situations. They begin to develop meaning for the four basic operations, with particular emphasis on addition and subtraction. Students learn a variety of strategies in developing meaning for basic addition and subtraction facts. They apply different methods of computing such as mental computation and paper-and-pencil algorithms.

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**Spatial Sense, Measurement, and Geometry**

Students learn the names and basic properties of simple geometric shapes, such as circles, squares, rectangles, triangles, spheres, cylinders, and cubes. They learn how shapes can be used to form patterns and tilings. They look for the shapes in nature and in objects people make, and practice drawing and using the shapes. They learn the meaning of basic directional and positional relationships such as near, far, inside, outside, and between.

They begin to understand the concepts of geometry and measurement by using a variety of manipulative materials to learn the difference between the length, width, and height of objects. As they learn about different tools for measuring, they describe, estimate, and measure length, weight, capacity, and temperature using non-standard and standard units. They begin to work with money, recognizing coins and their value. They use a variety of coins to represent different amounts of money. Students use the calendar to measure and model the days of the week and months of the year. They use clocks to explore and tell time to the nearest hour and half hour.

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**Patterns, Relationships, and Functions**

Young children learn about patterns by describing objects by their attributes. They compare, sort, and order things by one or more characteristics. They extend their understanding by finding and creating patterns, by correcting errors in patterns, and by translating patterns into different forms. Students also gather data pertaining to interests, family, and other things around them. They begin to understand that a number is a symbol for how much of something there is and begin to explore the use of a variable or placeholder and open sentences to express relationships. They also begin to use patterns as a problem-solving strategy.

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**Data, Probability, and Statistics.**

Students are introduced to the process of statistical investigation. They identify what data are needed to solve problems, plan and implement strategies for obtaining these data; and collect data by counting, measuring, and conducting simple surveys and experiments. They also analyze data using a variety of formats (e.g., pictures and tallies) and describe data using informal language as well as selected standard terminology such as tally. They recognize relationships between sets of data and begin to observe that changes in quantities may be interdependent.

Students conduct simple probability experiments using the process of statistical investigation. After gathering data, they interpret results in light of the likelihood of outcomes related to the occurrence of simple events. In investigating statistics and probability, students are introduced to counting strategies such as making tallies that reflect their beginning work in discrete mathematics.

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# Kindergarten

**Major Concepts**

- Read, write, and count using whole numbers
- Recognize circles, squares, triangles, and rectangles
- Identify and describe patterns
- Collect data and create graphs

## Number Sense, Numeration, and Numerical Operations

***Goal 1: The learner will recognize, model, and write numbers through 10.***

1.1 Model numbers in a variety of ways.

1.2 Read, write and count using whole numbers; rote count forward to 30 or beyond and backward from 10.

1.3 Use 1-1 correspondence to identify how many (0 - 10).

1.4 Recognize numerals and match to sets 0 - 10.

1.5 Write numerals 0-9 in meaningful contexts.

1.6 Use ordinals first through fifth.

1.7 Create and identify sets with more, less, or equal members by matching.

1.8 Combine and remove objects from sets, describe results.

1.9 Estimate quantities less than 20.

1.10 Create and solve story problems within a group.

1.11 Share equally (divide) between two people; explain solution.

## Spatial Sense, Measurement, and Geometry

***Goal 2: The learner will explore concepts of geometry and non-standard measurement.***

2.1 Recognize basic two-dimensional (plane) figures: circle, square, triangle, and rectangle. Describe their likenesses and differences and identify them in the environment.

2.2 Complete simple spatial visualization tasks and puzzles.

2.3 Compare and order objects using appropriate vocabulary.

2.4 Model and use directional and positional words.

2.5 Use non-standard measurement of length, weight, capacity, and time.

2.6 Name the days of the week.

## Patterns, Relationships, and Functions

*Goal 3: The learner will model simple patterns and sorting activities.*

3.1 Describe likenesses and differences between and among objects.

3.2 Sort by a given attribute; sort by own rule and explain.

3.3 Identify, copy, continue, and describe patterns.

3.4 Create patterns with actions, words and objects.

## Data, Probability, and Statistics

*Goal 4: The learner will gather and organize data in a group setting.*

4.1 Collect data to create concrete and pictorial graphs and describe the results as a group activity.

# Grade 1

| Major Concepts | Computational Skills to Maintain |
|---|---|
| • Addition and subtraction<br>• Spheres, cubes, cylinders, and cones<br>• Time<br>• Describe and compare objects<br>• Patterns<br>• Gather, organize, and display data | • Read, write and count using whole numbers |

## Number Sense, Numeration, and Numerical Operations

***Goal 1: The learner will read, write, and model numbers through 100 and compute with whole numbers.***

1.1 Count using one-to-one correspondence to 30.
1.2 Rote count by 1's, 5's and 10's to 100; by 2's to 20.
1.3 Make sets and match numerals up to 30.
1.4 Compare and order sets and numerals up to 30.
1.5 Read and write numerals to 100.
1.6 Read number words zero to ten.
1.7 Use ordinal numbers first through tenth.
1.8 Group and count objects by 2's, 5's, and 10's.
1.9 Identify one more/less/before/after/between.
1.10 Identify equal and unequal numerals and sets.
1.11 Represent numbers in a variety of ways: using tallies, building models to 100.
1.12 Estimate quantities up to 30. Recognize when solutions to problems are reasonable.
1.13 Group objects into tens and ones, recognize models; record.
1.14 Model concept of addition; know the combinations for sums to 10.
1.15 Model concept of subtraction as take-away, comparison, and missing addends.
1.16 Model the division of sets into two, three or four equal parts; explain solution.
1.17 Relate addition and subtraction to symbolic notation and write equations.
1.18 Find sums and differences using counting strategies such as counting on and counting back.
1.19 Memorize addition and subtraction facts to 10.
1.20 Model 10 more/less to 100.
1.21 Model 2-digit addition/subtraction with multiples of 10 to 100.

1.22 Create and solve problems using addition and subtraction. Use problem-solving strategies: modeling with manipulatives, acting out, drawing, using diagrams; use calculators as appropriate. Explain solutions.

## Spatial Sense, Measurement, and Geometry

***Goal 2: The learner will recognize, describe and identify simple geometric shapes and forms, and exhibit skills in using measurement.***

2.1 Recognize, identify, and describe plane geometric figures: circle, square, triangle, rectangle.
2.2 Recognize plane geometric figures: hexagon, trapezoid, and parallelogram.
2.3 Recognize basic three-dimensional (solid) figures: sphere, cube, cylinder and cone.
2.4 Identify open and closed figures.
2.5 Use directional and positional words.
2.6 Describe and compare characteristics of geometric figures.
2.7 Identify equal and unequal measures and regions.
2.8 Divide regions into two, three, and four equal parts.
2.9 Use non-standard units to estimate and measure length, weight, and capacity; record results.
2.10 Use calendar language appropriately, e.g. seasons and months of the year, today, yesterday, tomorrow, next week, last month.
2.11 Tell time to nearest hour using digital and analog clocks.
2.12 Solve problems involving non-standard measurement and explain strategy.
2.13 Solve spatial visualization puzzles and tasks; use visual memory.

## Patterns, Relationships, and Functions

*Goal 3: The learner will demonstrate an understanding of classification, patterning, and seriation.*

3.1 Describe and compare objects by their attributes; order sets.

3.2 Sort a set of objects in more than one way; sort by own rules and explain.

3.3 Copy, continue, and record patterns with actions, words and objects; translate into other forms.

3.4 Create and record patterns. Identify and name the pattern unit or numerical sequence.

3.5 Solve problems by identifying and correcting errors in repeating patterns.

3.6 Identify patterns in the environment.

## Data, Probability, and Statistics.

***Goal 4: The learner will demonstrate an understanding of data collection, display, and interpretation.***

4.1 Gather, organize and display information as a group activity.

4.2 Answer questions about charts and graphs.

4.3 Make predictions based on experiences.

4.4 Create concrete, pictorial, and symbolic graphs using prepared grids.

# Grade 2

| Major Concepts | Computational Skills to Maintain |
|---|---|
| • Place value<br>• Addition of multi-digit numbers<br>• Length, capacity, and weight<br>• Time and money<br>• Patterns<br>• Organization of data<br>• Simple probability experiments<br>• Students will create and solve relevant and authentic problems using appropriate technology and applying these concepts as well as those developed in previous years. | • Read, write and count using whole numbers<br>• Count using one-to-one correspondence<br>• Addition and subtraction facts |

## Number Sense, Numeration, and Numerical Operations

***Goal 1: The learner will read, write, and model numbers through 1000, and compute with numbers less than 1000.***

1.1 Rote count up to 1000.
1.2 Identify and use 10 more and 10 less.
1.3 Compare and order numbers; identify missing numbers in a sequence to 100.
1.4 Read w.rd names for numbers to 100.
1.5 Use counting strategies such as skip counting by 2's, 5's, and 10's and grouping objects by 3's and 4's.
1.6 Identify odd and even numbers using objects.
1.7 Group objects into ones, tens, and hundreds and record in standard form.
1.8 Model 3-digit numbers; identify, read, and write correct numerals.
1.9 Indicate the value of each digit in any 2 or 3-digit number.
1.10 Use problem-solving strategies such as diagrams, organized lists, manipulatives, act out, guess and check, pictures; use calculators when appropriate.
1.11 Explain solutions to problems using words, pictures, and numbers.
1.12 Make reasonable estimates up to 100 objects.
1.13 Identify missing addends for addition facts to 18.
1.14 Add 3 single-digit numbers.
1.15 Model 2-digit addition and subtraction using manipulatives and alternative strategies; record, and explain.
1.16 Memorize addition/subtraction facts up to 18.
1.17 Add 2- and 3- digit numbers with and without regrouping.

1.18 Use addition/subtraction strategies to solve problems.

1.19 Divide regions/sets into halves, thirds, and fourths. Record in fractional form.

1.20 Model repeated addition (multiplication) and sharing equally (division); record solutions.

## Spatial Sense, Measurement, and Geometry

***Goal 2: The learner will recognize, understand, and use basic geometric properties, and standard units of metric and customary measurement.***

2.1 Describe and make plane figures: squares, rectangles, triangles, circles, hexagons, trapezoids, and parallelograms.

2.2 Describe and make solid figures: cubes, rectangular prisms, spheres, cylinders, cones, and pyramids.

2.3 Identify and make figures with line symmetry.

2.4 Identify and make congruent figures.

2.5 Use spatial visualization to solve problems; demonstrate visual memory.

2.6 Measure lengths in inches/centimeters; record results.

2.7 Measure capacity to the nearest cup/liter; record results.

2.8 Weigh objects to the nearest pound/kilogram; record results.

2.9 Read Fahrenheit thermometers in increments of 1's, 2's, and 5's; record results.

2.10 Sequence months; use the calendar to solve problems.

2.11 Tell time to the nearest half-hour using digital and analog clocks; record. Solve problems related to time.

2.12 Determine the value of sets of coins (pennies, nickels, dimes, quarters); record using appropriate notation.

2.13 Make different sets of coins with equivalent values.

2.14 Identify coins needed to buy items priced at $1.00 or less.

2.15 Solve problems using money. Estimate costs and make change using coins up to $1.00.

## Patterns, Relationships, and Functions

*Goal 3: The learner will demonstrate an understanding of classification, patterning, and seriation.*

3.1 Sort by one or more attributes; describe rules used.
3.2 Identify classification and patterning in the environment.
3.3 Define, continue, and describe rules for geometric patterns.
3.4 Use patterns to continue numerical sequences; identify the rule.
3.5 Identify and correct errors in numerical and geometric patterns.
3.6 Solve simple logic problems.
3.7 Define and continue pattern units; translate into other forms.

## Data, Probability, and Statistics

*Goal 4: The learner will demonstrate an understanding of data collection, display, and interpretation.*

4.1 Collect, sort, organize, and display information in charts, graphs, and tables with correct labeling.
4.2 Summarize and interpret information in charts, graphs, and tables; make predictions.
4.3 Collect and display data over a period of time.
4.4 Locate points on the number line and positions on a grid.
4.5 Complete simple probability experiments; describe results and make predictions.

## Intermediate Grades 3-5

**Number Sense, Numeration, and Numerical Operations**

Youngsters in grades 3-5 represent whole numbers, fractions, and decimals with concrete objects, pictures, and symbols in a variety of contexts. They recognize equivalent fractions and decimals and explain the basis for the equivalence. They compare and order fractions and decimals. They understand and use the place value system and various properties of numbers, including the properties of special numbers like 0 and 1; round numbers to a specified precision; and reasonably estimate answers to computations.

Students use a variety of tools to model operations with whole numbers, develop methods for recording operations, and relate models to standard symbolic expressions and algorithms. They further develop and apply different methods of computing, such as mental computation, paper-and-pencil algorithms, and technology. They learn the fundamental order of operations and explore various properties of operations.

Students use a variety of strategies for learning basic multiplication and division facts. They explain why such strategies work by modeling using counters or other tools. Overall, students have a firm foundation in the understanding of place value, quickly recall basic addition, subtraction, multiplication, and division facts, and easily carry out operations with whole numbers.

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**Spatial Sense, Measurement, and Geometry**

Children in grades 3-5 measure distance, area, and capacity in both customary and metric units. They tell and write time using digital and analog clocks. Using clocks and calendars they explore elapsed time problems. They explore concepts of perimeter and area of rectangles and squares and develop the basic formulas for computing these quantities. They approximate or measure the areas of irregular figures by covering them with simpler figures such as squares or triangles. They estimate distances and draw and build plane and solid figures. They learn about angle measurement, basic properties of circles, and classification of polygons and polyhedra. They investigate the basic geometric relationships, such as parallel, perpendicular, congruent, and similar. They recognize symmetry and geometric transformations. They plot points on a rectangular grid and read graphs drawn on rectangular grids.

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**Patterns, Relationships, and Functions**

In grades 3-5, students continue to identify and describe patterns in many situations, including patterns in real-world data. They use multiple tools such as concrete materials, calculators, and computers to investigate and discover patterns. Patterns are used in geometry and other mathematics to develop new concepts. Students make tables and graphs to show relationships and then verbally describe the patterns. They use patterns to extend their data, generate a rule from the relationship, and make predictions. They begin to understand that symbols can be used to stand for unknown quantities and that these symbols can be used in expressions, in open sentences, and in describing relationships. They begin to develop original expressions verbally and algebraically.

**Data, Probability, and Statistics.**

Students continue working with the process of statistical investigation. Techniques for data collection become more sophisticated, for example, informal consideration of sample size and collection of data over time. The nature and kinds of representations used include tables and graphs such as bar, circle and pictograph. In addition, students use coordinate graphs to explore relationships among pairs of data. Students describe data using standard measures such as median, mode, and range and are introduced to the concept of mean.

Applying the process of statistical investigation, students conduct probability experiments, recording the ways in which the conditions of simple experiments affect the outcomes. The language of simple fractions is used to compare probabilities. In investigating statistics and probability, students extend their work with counting strategies, exploring the multiplicative nature of working with simple combinations.

# Grade 3

| Major Concepts | Computational Skills to Maintain |
|---|---|
| • Multiplication facts/tables<br>• Subtraction of multi-digit numbers<br>• Length, capacity, and weight<br>• Time and temperature<br>• Polygons and polyhedra<br>• Patterns<br>• Read and interpret graphs<br>• Permutations and combinations<br>• Students will create and solve relevant and authentic problems using appropriate technology and applying these concepts as well as those developed in previous years. | • Count using one-to-one correspondence<br>• Addition and subtraction facts<br>• Use counting strategies<br>• Add multi-digit numbers |

## Number Sense, Numeration, and Numerical Operations

***Goal 1: The learner will model, identify and compute with numbers less than 10,000.***

1.1 Read and write word names for numbers to 1,000.

1.2 Relate standard and expanded notation to 3- and 4-digit numbers.

1.3 Compare and order numbers less than 10,000.

1.4 Use estimation techniques in determining solutions to problems.

1.5 Identify odd and even numbers; generalize ways to determine odd or even.

1.6 Model fractions and mixed numbers using regions and sets; describe relationships of parts to whole; record.

1.7 Compare and order fractions using models; describe comparisons.

1.8 Model equivalent fractions using manipulatives and pictures.

1.9 Subtract 2- and 3-digit numbers.

1.10 Model and explain multiplication in a variety of ways including repeated addition, rectangular arrays, and skip counting.

1.11 Model and use the identity and commutative properties for addition and multiplication.

1.12 Model and explain division in a variety of ways including sharing equally, repeated subtraction, rectangular arrays, and its relationship to multiplication.

1.13 Memorize multiplication facts/tables through 10.

1.14 Determine if there is sufficient information to solve a problem; identify missing or extraneous data in problem-solving situations.

1.15 Solve meaningful, multi-step problems involving addition, subtraction and multiplication using a variety of strategies; use calculators as appropriate

## Spatial Sense, Measurement , and Geometry

*Goal 2: The learner will recognize, understand, and use basic geometric properties, and standard units of metric and customary measurement.*

2.1 Draw and classify polygons and polyhedra (solid figures) using appropriate vocabulary: faces, angles, edges, and vertices. Describe the rules for grouping.

2.2 Identify and model symmetry and congruence with concrete materials and drawings.

2.3 Construct with cubes a solid to match a given picture or model.

2.4 Recognize a three-dimensional object from different perspectives.

2.5 Observe and describe geometry in the environment.

2.6 Estimate and measure length (inches, feet, yards, centimeters, meters), weight (grams, ounces, pounds), and capacity (cups, pints, quarts, gallons, liters) using appropriate tools and units.

2.7 Model and compare units within the same measurement system.

2.8 Model the concepts of area and perimeter using concrete materials, non-standard, and standard units. Estimate, record, and explain results.

2.9 Determine the value of sets of coins to $5.00 and create equivalent amounts with different coins.

2.10 Estimate and compute the cost of items up to $5.00; make change up to $5.00.

2.11 Tell time to the nearest minute with digital and analog clocks; record. Solve problems related to time.

2.12 Read Celsius and Fahrenheit thermometers; relate temperatures to everyday situations.

2.13 Solve problems using measurement concepts and procedures. Explain the solutions

## Patterns, Relationships, and Functions

***Goal 3: The learner will demonstrate an understanding of classification, patterning, and seriation.***

3.1 Organize objects or ideas into groups; describe attributes of groups and rules for sorting.

3.2 Describe and demonstrate patterns in skip counting and multiplication; continue sequences beyond memorized or modeled numbers.

3.3 Extend and create geometric and numeric sequences; describe patterns in a variety of ways; use calculators and computers where appropriate.

3.4 Analyze patterns; describe properties and translate into different forms. Create and record similar patterns.

3.5 Use patterns to make predictions and solve problems.

3.6 Use Venn diagrams as a problem-solving strategy to illustrate similarities and differences in sets.

## Data, Probability, and Statistics

***Goal 4: The learner will demonstrate an understanding of data collection, display, and interpretation.***

4.1 Gather and organize data from surveys and classroom experiments, including data collected over a period of time.

4.2 Display data on charts and graphs: picture, bar and line plots; describe data using mode.

4.3 Construct graphs where symbols or scales represent multiple units.

4.4 Read and interpret graphs and charts (bar, picture, circle, line and line plots) as sources of information; identify main idea, draw conclusions and make predictions.

4.5 Name the ordered pair for a point on the grid; plot positions named by ordered pairs on a coordinate grid.

4.6 Construct and use time lines to display sequences of events.

4.7 Describe the probability of chance events as more, less or equally likely to occur.

4.8 List arrangements (permutations) and combinations of up to three items.

# Grade 4

| Major Concepts | Computational Skills to Maintain |
|---|---|
| • Addition, subtraction, and multiplication with multi-digit numbers<br>• Division with single digit divisors<br>• Points, lines, angles, and transformations in geometry<br>• Non-numeric symbols to represent quantities<br>• Range, median, and mode<br>• Bar, picture, and circle graphs; stem-and-leaf plots and line plots<br>• Probability<br>• Students will create and solve relevant and authentic problems using appropriate technology and applying these concepts as well as those developed in previous years. | • Use counting strategies<br>• Add and subtract multi-digit numbers<br>• Read and write word names for numbers<br>• Addition, subtraction, multiplication facts/ tables<br>• Identify, explain, and apply the commutative and identity properties for multiplication and addition |

## Number Sense, Numeration, and Numerical Operations

***Goal 1: The learner will read, write, model, and compute with rational numbers.***

1.1 Read and write numbers less than one million using standard and expanded notation.

1.2 Use estimation techniques in determining solutions to problems.

1.3 Model and identify the place value of each digit in a multi-digit numeral to the hundredths place.

1.4 Model, identify, and compare rational numbers (fractions and mixed numbers).

1.5 Identify and compare rational numbers in decimal form (tenths and hundredths) using models and pictures.

1.6 Relate decimals and fractions (tenths and hundredths) to each other using models and pictures.

1.7 Use models and pictures to add and subtract decimals, explaining the processes and recording results.

1.8 Use models and pictures to add and subtract rational numbers with like denominators.

1.9 Find the fractional part of a whole number using models and pictures.

1.10 Model and explain associative and distributive properties.

1.11 Memorize the division facts related to the multiplication facts/tables through 10.

1.12 Identify missing factors in multiplication facts.

1.13 Round rational numbers to the nearest whole number and justify.

1.14 Estimate solutions to problems.

1.15 Multiply 2- or 3- digit numbers by 1-digit numbers or a 2-digit multiple of 10.

1.16 Divide using single-digit divisors, with and without remainders.

1.17 Use order of operations with addition, subtraction, multiplication, and division.

1.18 Solve multi-step problems; determine if there is sufficient data given, then select additional strategies including:

- make a chart or graph
- look for patterns
- make a simpler problem
- use logic
- work backwards
- break into parts.

Verify and interpret results with respect to the original problem; use calculators as appropriate. Discuss alternate methods for solution.

## Spatial Sense, Measurement, and Geometry

***Goal 2: The learner will demonstrate an understanding and use of the properties and relationships in geometry, and standard units of metric and customary measurement.***

2.1 Identify points, lines, and angles (acute, right, and obtuse); identify in the environment.

2.2 Use manipulatives, pictorial representations, and appropriate vocabulary (e.g. sides, angles, and vertices) to identify properties of plane figures; identify in the environment.

2.3 Use manipulatives, pictorial representations, and appropriate vocabulary (e.g. faces, edges, and vertices) to identify properties of polyhedra (solid figures); identify in the environment.

2.4 Identify intersecting, parallel, and perpendicular lines and line segments and their midpoints; identify in the environment.

2.5 Recognize congruent plane figures after geometric transformations such as rotations (turns), reflections (flips), and translations (slides).

2.6 Use designs, models, and computer graphics to illustrate reflections, rotations, and translations of plane figures and record observations.

2.7 Estimate and measure length, capacity and mass using these additional units: inches, miles, centimeters, and kilometers; milliliters, cups, and pints; kilograms and tons.

2.8 Write and solve meaningful, multi-step problems involving money, elapsed time, and temperature; verify reasonableness of answers.

2.9 Use models to develop the relationship between the total number of square units contained in a rectangle and the length and width of the figure.

2.10 Measure the perimeter of rectangles and triangles. Determine the area of rectangles and squares using grids; find areas of other regular and irregular figures using grids.

## Patterns, Relationships, and Functions

*Goal 3: The learner will demonstrate an understanding of patterns and relationships.*

3.1 Identify numerical and geometric patterns by stating their rules; extend the pattern, generalize, and make predictions.

3.2 Identify the pattern by stating the rule, extend the pattern, generalize the rule for the pattern, and make predictions when given a table of number pairs or a set of data.

3.3 Construct and order a table of values to solve problems associated with a given relationship.

3.4 Use non-numeric symbols to represent quantities in expressions, open sentences, and descriptions of relationships. Determine solutions to open sentences.

## Data, Probability, and Statistics

*Goal 4: The learner will demonstrate an understanding and use of graphing, probability, and data analysis.*

4.1 Interpret and construct stem-and-leaf plots.

4.2 Display data in a variety of ways including circle graphs. Discuss the advantages and disadvantages of each form including ease of creation and purpose of the graph.

4.3 Collect, organize, and display data from surveys, research, and classroom experiments, including data collected over time. Include data from other disciplines such as science, physical education, social studies, and the media.

4.4 Interpret information orally and in writing from charts, tables, tallies, and graphs.

4.5 Use range, median, and mode to describe a set of data.

4.6 Plot points that represent ordered pairs of data from many different sources such as economics, science experiments, and recreational activities.

4.7 Investigate and discuss probabilities by experimenting with devices that generate random outcomes such as coins, number cubes, spinners.

4.8 Use a fraction to describe the probability of an event and report the outcome of an experiment.

# Grade 5

| Major Concepts | Computational Skills to Maintain |
|---|---|
| • Division with multi-digit divisors<br>• Addition, subtraction, and multiplication of fractions<br>• Add, subtract, compare, and order decimals<br>• Area and perimeter<br>• Circles<br>• Draw and measure angles<br>• Algebraic expressions<br>• Central tendency-mean<br>• Line graphs<br>• Experimental and theoretical probabilities<br>• Students will create and solve relevant and authentic problems using appropriate technology and applying these concepts as well as those developed in previous years. | • Read and write word names for numbers<br>• Multiplication facts/tables<br>• Division facts<br>• Add and subtract multi-digit numbers<br>• Estimate products; multiply multi-digit numbers<br>• Divide using single digit divisors<br>• Use order of operations<br>• Identify, explain, and apply the commutative, associative, distributive and identity properties |

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will understand and compute with rational numbers.***

1.1 Use place value through millions in real-world situations including reading, writing, estimating, and comparing numbers in a variety of forms.

1.2 Estimate products; multiply any whole number by a 2- or 3-digit factor.

1.3 Relate exponential notation to repeated multiplication.

1.4 Estimate and solve division problems with 2- and 3- digit divisors; explain solution.

1.5 Use the order of operations to simplify numerical expressions.

1.6 Find multiples, common multiples, and least common multiple of numbers; explain.

1.7 Find the factors, common factors, and greatest common factor of numbers; explain.

1.8 Identify prime and composite numbers less than 100.

1.9 Identify equivalent decimals and fractions at the symbolic level. Explain the equivalence.

1.10 Compare and order numbers with decimals to the thousandths place; explain solution.

1.11 Compare and order fractions which are given with the same numerators or the same denominators; explain solution.

1.12 Add and subtract fractions with like denominators.

1.13 Multiply a fraction by a whole number.

1.14 Use models and pictures to add and subtract fractions and mixed numbers with unlike denominators; record solutions.

1.15 Estimate results and compute sums and differences with decimal numbers.

1.16 Use models and pictures to multiply a whole number by a decimal number; record and explain.


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1.17 Determine if there is sufficient information to solve a problem; identify missing or extraneous data in problem-solving situations.

1.18 Solve multi-step problems using an organized approach, and selecting additional strategies including

- restate the problem
- classify
- lists
- write a number sentence

Verify and interpret results with respect to the original problem; use calculators as appropriate.

## Spatial Sense, Measurement, and Geometry

***Goal 2: The learner will demonstrate an understanding and use of the properties and relationships in geometry, and standard units of metric and customary measurement.***

2.1 Use and make models to demonstrate formulas for the area and perimeter of squares and rectangles, to compare units of area within the same system, and to investigate and compare units of volume.

2.2 Calculate the area and perimeter of rectangles and the perimeters of plane figures.

2.3 Use concrete and pictorial representations and appropriate vocabulary to compare and classify polygons and polyhedra; create models of polyhedra (cubes, cylinders, cones prisms, and pyramids.)

2.4 Use a compass to draw circles; identify and determine the relationships among the radius, diameter, chord, center, and circumference.

2.5 Use a protractor to draw and measure acute, right, and obtuse angles; identify and label the vertex, rays, interior and exterior of an angle.

2.6 Use a variety of quadrilaterals and triangles to draw conclusions about the sum of the measures of the interior angles; use appropriate technology.

2.7 Model proportions by reducing or enlarging drawings using grids.

2.8 Investigate similar figures using rulers and protractors.

2.9 Use an organized approach, appropriate strategies, and technology as needed to solve multi-step problems involving geometry, spatial visualization, and measurement (length, weight, time, capacity, temperature, perimeter, area, volume.)

2.10 Verify and interpret results with respect to the original problem; identify alternate strategies for solving a problem. Use calculators and computers as appropriate.

## Patterns, Relationships, and Functions

*Goal 3: The learner will demonstrate an understanding of patterns, relationships, and elementary algebraic representation.*

3.1 Investigate patterns that occur when changing numerators or denominators of fractions. Model with concrete materials and extend to calculator investigations.

3.2 Identify and use the rules for divisibility.

3.3 Use patterns, relationships, and functions occurring in computation, geometry, graphs, and other applications to make generalizations and predict results.

3.4 Use models to represent variables, expressions, and relationships.

3.5 Use an organized approach and appropriate strategies including calculators to solve multi-step problems involving patterns, relationships, and functions.

## Data, Probability, and Statistics

*Goal 4: The learner will demonstrate an understanding and use of graphing, probability and data analysis.*

4.1 Interpret and construct line graphs.

4.2 Explain the kinds of decisions that need to be made in selecting and constructing appropriate graphs including pictograph, bar, line plot, circle, and line graph.

4.3 Systematically collect, organize, display and interpret data both orally and in writing using information from a variety of content areas.

4.4 Compare increasingly complex displays of data, including multiple sets of data on the same graph, computer applications, and Venn diagrams

4.5 Determine the mean of a given set of data using a calculator when appropriate.

4.6 Use the range, median, mean and mode to describe a set of data.

4.7 Show all arrangements (permutations) and combinations of up to four items; list and explain all possible outcomes in a given situation.

4.8 Compare experimental and theoretical (expected) results for a variety of simple experiments.

4.9 Use an organized approach and appropriate strategies to solve multi-step problems involving graphing, probability, and statistics. Use calculators and computers as appropriate.

# Middle Grades 6-8

**Number Sense, Numeration, and Numerical Operations**

Students in the middle years represent integers, rational numbers, and irrational numbers using concrete objects, pictures, and symbols in a variety of contexts. They explore relationships among rational numbers and recognize equivalence for fractions, decimals, and percents and explain the basis for the equivalence. They extend understanding of place value to decimal and scientific notation. They recognize properties of integers, rational, and some real numbers, including 0,1, and inverses. They express numerical comparisons as ratios and rates and solve problems using ratio, proportion, and percent.

Students understand number theory relationships including prime and composite numbers, factors, and multiples. They explain the meaning of powers and square roots of numbers, develop facility with estimation and mental computation involving square numbers, use calculators to compute powers and roots, and can apply the laws of exponents in problem situations.

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**Spatial Sense, Measurement, and Geometry**

Students expand their study of perimeter and area of parallelograms, triangles, and circles and develop the basic formulas for computing these quantities. They extend this study to an exploration of surface area and volume of prisms, cylinders and cones. Students use computers, calculators, and other tools to draw and construct figures and investigate their properties. They use and make a variety of geometric models in solving problems, becoming proficient in interpreting problem situations geometrically. They explore the concept of slope and relate it to linear variation and rates of change in applications. They plot lines and curves in rectangular coordinates, using pencil and paper, graphing calculators, and computer software.

They study congruent and similar triangles and use the Pythagorean Theorem. They continue to study symmetries and transformations and become proficient at visualizing and recognizing figures which have been changed by slides, rotations and/or reflections. Students measure to specified levels of precision and assess errors in measurement. They study the relationships among length, perimeter, area, and volume.

Students draw or construct figures and investigate their properties using computer software as well as standard drafting tools. They also draw three-dimensional figures from different perspectives. They use ratio and proportion in the study of similar figures. They calculate distances and areas from scale drawings and maps and study measurement systems.

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**Patterns, Relationships, and Functions**

Students identify patterns and relationships in context, symbolize them, and express them algebraically. They use concrete materials to develop concepts of operations with variables. They use variables to show interdependence of quantities and to explore questions of what happens to one quantity or variable as the other quantity or variable changes. Formulas provide a context for students to examine and investigate this interdependence. Students gather ordered pairs of linear data, describe the linear relationship between the variables, and represent the relationship in symbols. Students begin to use the language of functions. From tables and graphs students recognize nonlinear relationships and functions. Students increase their understanding of variable through the use of technologies such as graphing calculators and spreadsheets. They write equations for relationships and then graph the equations using a graphing calculator to analyze a situation or make predictions. They solve linear equations and inequalities using multiple strategies including use of concrete materials, tables, guess-and-test, working backwards, and algebraic methods. Students simplify algebraic expressions involving numbers and variables and apply algebraic methods to solve a variety of real-world and mathematical problems.

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**Data, Probability, and Statistics.**

Students investigate more complex data sets using technologies such as spreadsheets, data bases, and graphing calculators. They use more complex representations, such as histograms, box plots, and scatter plots, which highlight an increased understanding of spread and grouping of data and the relationships between variables. They identify basic patterns and trends in tables and charts and use these patterns and trends to make predictions. They describe the distribution of data using measures of central tendency such as mean and measures of spread such as range. Students informally consider topics such as extremes in the data, representativeness, and misuse of representations to communicate information. With bivariate data, they learn to recognize relationships, estimate, and make predictions. They learn that the analysis of a well-chosen representative sample can yield useful information about an entire population, and they begin to choose samples that are sufficiently large and free from bias.

Continuing to use the process of statistical investigation, students conduct experiments and simulations to investigate basic properties of probability, including dependent and independent events. They use a variety of representations, including bar graphs, histograms, and line graphs to display their results. They compare their experimental probabilities with probabilities they have derived theoretically and learn that the level of agreement between the two should depend on the number of times an experiment is repeated. They learn to make inferences and predictions based on the outcomes of their experiments and simulations.

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# Grade 6

| Major Concepts | Computational Skills to Maintain |
|---|---|
| • Computation with fractions, mixed numbers, and decimals<br>• Compare and order fractions, decimals, percents and integers<br>• Area formulas for triangles, parallelograms, and circles<br>• Relations and functions<br>• Analyze data<br>• Coordinate graphing<br>• Probability of simple events<br>• Students will create and solve relevant and authentic problems using appropriate technology and applying these concepts as well as those developed in previous years. | • Multiplication and division facts<br>• Estimate results and apply all operations with whole numbers<br>• Use order of operations<br>• Find the factors, common factors, and greatest common factor of numbers<br>• Compute with fractions<br>• Add and subtract with decimal numbers<br>• Identify, explain, and apply the commutative, associative, distributive and identity properties |

## Number Sense, Numeration, and Numerical Operations

***Goal 1: The learner will understand and compute with rational numbers.***

1.1 Read, write and make models of numbers including percents and exponentials.

1.2 Relate fractions, decimals, and percents.

1.3 Compare and order fractions, decimals, and percents.

1.4 Multiply and divide fractions, mixed numbers, and decimals using models and pictures; record solution.

1.5 Multiply and divide fractions, mixed numbers, and decimals.

1.6 Add and subtract fractions and mixed numbers with unlike denominators.

1.7 Use estimation and mental math to solve problems with fractions, decimals, and percents; explain solution.

1.8 Solve problems using prime factorization, common factors and common multiples. Explain solutions.

1.9 Use models and pictures to relate concepts of ratio, proportion, and percent; record results.

1.10 Use models and pictures to demonstrate understanding of integers. Record results.

1.11 Compare and order integers.

1.12 Use the order of operations to simplify numerical expressions with parentheses and exponents.

1.13 Translate word problems into number sentences and solve. Explain solutions.

1.14 Analyze problem situations, determine if there is sufficient information to solve the problem, identify missing or extraneous data, select appropriate strategies, and use an organized approach to solve multi-step problems; use calculators when appropriate.

## Spatial Sense, Measurement, and Geometry

***Goal 2: The learner will demonstrate an understanding and use of the properties and relationships in geometry, and standard units of metric and customary measurement.***

2.1 Construct congruent segments, congruent angles, bisectors of line segments and bisectors of angles.

2.2 Define and identify interior, exterior, complementary, and supplementary angles and pairs of lines including skew lines.

2.3 Define and identify alternate interior, alternate exterior, corresponding and vertical angles.

2.4 Identify and distinguish among similar, congruent and symmetric figures; name corresponding parts.

2.5 Locate, give the coordinates of, and graph plane figures which are the results of translations or reflections in the first quadrant.

2.6 Investigate and determine the relationship between the diameter and circumference of a circle and the value of pi; calculate the circumference of a circle.

2.7 Identify the relationship between areas of triangles and rectangles with the same base and height.

2.8 Use models to develop formulas for finding areas of triangles, parallelograms and circles.

2.9 Calculate areas of triangles, parallelograms and circles.

2.10 Model the concept of volume for rectangular solids as the product of the area of the base and the height.

2.11 Convert measures of length, area, capacity, weight and time expressed in a given unit to other units in the same measurement system.

2.12 Estimate solutions to problems involving geometry and measurement. Determine when estimates are sufficient for the measurement situation.

2.13 Analyze problem situations, select appropriate strategies, and use an organized approach to solve non-routine and increasingly complex problems involving geometry and measurement. Use technology as appropriate.

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## Patterns, Relationships, and Functions

***Goal 3: The learner will demonstrate an understanding of patterns, relationships, and algebraic representations.***

3.1 Describe, extend and write rules for a variety of patterns.

3.2 Generate a set of ordered pairs using a given rule which is stated verbally or algebraically.

3.3 Given a group of ordered pairs, identify either verbally or algebraically the rule used to generate them and record results.

3.4 Use variables to describe numerical expressions and relationships.

3.5 Use graphs and tables to represent ordered pairs; describe the relationship; recognize both linear and nonlinear relationships.

3.6 Identify and use patterning as a strategy to solve problems.

## Data, Probability, and Statistics

***Goal 4: The learner will demonstrate an understanding and use of graphing, probability, and data analysis.***

4.1 Create and evaluate graphic representations of data.

4.2 Analyze data using spreadsheets.

4.3 Locate points in all quadrants of the coordinate plane using ordered pairs.

4.4 Use measures of central tendency to compare two sets of data.

4.5 Construct convincing arguments based on analysis of data and interpretation of graphs.

4.6 Design an experiment to test a theoretical probability; record and explain results.

4.7 Make predictions based on the probabilities of simple events.

4.8 Use inductive and deductive reasoning to solve problems.

4.9 Analyze problem situations, use an organized approach, and select appropriate strategies and technology to solve problems involving probability and statistics.

# Grade 7

| Major Concepts | Computational Skills to Maintain |
|---|---|
| • Compute with integers<br>• Ratio, proportion, and percent<br>• Compare and order rational numbers<br>• Geometric transformations in the coordinate plane<br>• Proportional relationships and similar figures<br>• Volume<br>• Simple linear equations and inequalities<br>• Histograms<br>• Probability of independent events<br>• Analyze graphic representations of data<br>• Students will create and solve relevant and authentic problems using appropriate technology and applying these concepts as well as those developed in previous years. | • Estimate products; multiply with multi-digit factors<br>• Estimate quotients; divide with 2- and 3- digit divisors<br>• Use order of operations<br>• Find the factors, common factors, and greatest common factor of numbers<br>• Compute with whole numbers, decimals, and fractions<br>• Identify, explain, and apply the commutative, associative, distributive and identity properties |

## Number Sense, Numeration and Numerical Operations

***Goal 1: The learner will understand and compute with real numbers.***

1.1 Write whole numbers in scientific notation; convert scientific notation to standard form; investigate the uses of scientific notation.

1.2 Compare and order rational numbers.

1.3 Model addition, subtraction, multiplication, and division of integers; record.

1.4 Compute with integers.

1.5 Write and solve proportions.

1.6 Estimate and solve problems using ratio, proportion and percent including discounts, taxes, commissions, and simple interest.

1.7 Use geometric models to develop the meaning of the square of a number and its positive square root; investigate and estimate square root, checking the results with a calculator.

1.8 Analyze and select appropriate operations, models, strategies and methods to solve a variety of multi- step problems using positive rational numbers, integers, and their inverses. Use calculators and computers where appropriate.

## Spatial Sense, Measurement and Geometry

***Goal 2: The learner will demonstrate an understanding and use of the properties and relationships in geometry, and standard units of metric and customary measurement.***

2.1 Construct perpendicular and parallel lines.

2.2 Identify the congruent and supplementary relationships of the angles formed by cutting parallel lines by a transversal.

2.3 Locate, give the coordinates of, and graph plane figures which are the results of translations or reflections in all quadrants of the coordinate plane.

2.4 Use models to investigate the concept of the Pythagorean Theorem.

2.5 Build models of three-dimensional figures given end, side and top views.

2.6 Draw end, side and top views of three-dimensional figures given models; use appropriate technology.

2.7 Use models to find the surface area of rectangular solids and cylinders.

2.8 Use models to find the volume of prisms and cylinders.

2.9 Calculate the volume of rectangular solids.

2.10 Recognize the effect on the area and perimeter when one or two dimensions of a plane figure are changed.

2.11 Use proportions to express relationships between corresponding parts of similar figures.

## Patterns, Relationships and Functions

*Goal 3: The learner will demonstrate an understanding of patterns, relationships, and fundamental algebraic concepts.*

3.1 Evaluate algebraic expressions.

3.2 Model and solve simple equations and inequalities and graph their solutions; use appropriate technology.

3.3 Write or model a simple linear equation or inequality to solve a given problem; use appropriate technology.

3.4 Write a problem given a simple linear equation or inequality.

3.5 Describe, extend, analyze and create a wide variety of patterns to investigate relationships and solve problems; use appropriate technology.

## Data, Probability, and Statistics

*Goal 4: The learner will demonstrate an understanding and use of graphing, probability, and data analysis.*

4.1 Interpret and construct histograms.

4.2 Compare and relate bar graphs and histograms.

4.3 Construct circle graphs using ratios, proportions, and percents.

4.4 Create, compare, contrast, and evaluate both orally and in writing, different graphic representations of the same data.

4.5 Identify appropriate uses of different measures of central tendency.

4.6 Recognize and identify misuses of statistical and numerical data.

4.7 Find all possible outcomes of simple experiments using such methods as lists, tree diagrams, frequency distribution tables, and the Fundamental Counting Principle.

4.8 Compute and apply simple permutations and combinations.

4.9 Find the probability of independent events.

4.10 Identify/explain the relationship between experimental results and theoretical probability.

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# Grade 8

| Major Concepts | Computational Skills to Maintain |
|---|---|
| • Computation with rational numbers<br>• Conversions among fractions, decimals, and percents<br>• Pythagorean Theorem<br>• Surface area and volume<br>• Linear equations and inequalities<br>• Box plots and scatter plots<br>• Probability of independent and dependent events<br>• Theoretical probabilities and experimental results<br>• Students will create and solve relevant and authentic problems using appropriate technology and applying these concepts as well as those developed in previous years. | • Compute with whole numbers, decimals, and fractions<br>• Use order of operations<br>• Compute with integers<br>• Identify, explain, and apply the commutative, associative, distributive, inverse, and identity properties |

## Number Sense, Numeration, and Numerical Operations

***Goal 1: The learner will understand and compute with real numbers.***

1.1 Identify subsets of the real number system.

1.2 Estimate and compute with rational numbers.

1.3 Compare, order, and convert among fractions, decimals (terminating and non-terminating), and percents.

1.4 Solve problems involving percent of increase and percent of decrease.

1.5 Use scientific notation to express large numbers and numbers less than one. Write in standard form numbers given in scientific notation.

1.6 Use rules of exponents.

1.7 Estimate the square root of a number between two consecutive integers; using a calculator, find the square root of a number to the nearest tenth.

1.8 Solve problems involving exponents and scientific notation.

1.9 Determine the absolute value of a number.

1.10 Identify, explain, and apply the commutative, associative, and distributive properties, inverses, and identities in algebraic expressions.

1.11 Simplify algebraic expressions.

1.12 Analyze problems to determine if there is sufficient or extraneous data, select appropriate strategies, and use an organized approach to solve using calculators when appropriate.

## Spatial Sense, Measurement, and Geometry

*Goal 2: The learner will demonstrate an understanding and use of the properties and relationships in geometry, and standard units of metric and customary measurement.*

2.1 Use geometric concepts and modeling to interpret and solve problems.

2.2 Calculate distances and areas from scale drawings and maps.

2.3 Find the surface area of rectangular solids and cylinders.

2.4 Use models to investigate the relationship of the volume of a cone to a cylinder and a pyramid to a prism with the same base and height.

2.5 Find the volume of prisms, cylinders, pyramids, and cones, with and without models.

2.6 Use the Pythagorean Theorem to solve problems.

2.7 Determine the effect on the volume of solid figures when one or more dimension is changed.

2.8 Solve problems related to similar and congruent figures.

2.9 Locate, give the coordinates of, and graph plane figures which are the results of rotations (multiples of 90°). Graph plane figures which are similar to a given figure (dilations.)

2.10 Identify and draw 3-dimensional figures from different perspectives (top, side, front, corner); use appropriate technology.

2.11 Build 3-dimensional figures given various views.

2.12 Select appropriate units and tools for measurement tasks within problem-solving situations; determine precision and check for reasonableness of results.

## Patterns, Relationships, and Functions

***Goal 3: The learner will demonstrate an understanding of patterns, relationships, and fundamental algebraic concepts.***

3.1 Use formulas in problem-solving situations.

3.2 Solve one and two-step linear equations and inequalities.

3.3 Graph a linear equation using ordered pairs. Investigate the graphs of linear inequalities; use appropriate technology.

3.4 Investigate the concept of slope; use appropriate technology.

3.5 Describe, extend, and analyze a wide variety of geometric and numerical patterns, such as Pascal's triangle or the Fibonacci sequence; use appropriate technology.

## Data, Probability, and Statistics

***Goal 4: The learner will demonstrate an understanding and use of graphing, probability, and data analysis.***

4.1 Interpret and construct box plots.

4.2 Collect data involving two variables and display on a scatter plot; interpret results; identify positive and negative relationships.

4.3 Interpret the mean, explain its sensitivity to extremes, and explain its use in comparison with the median and the mode.

4.4 Evaluate arguments based on data. Discuss random vs. biased sampling.

4.5 Find the probability of independent and dependent events.

4.6 Make predictions based on theoretical probabilities and experimental results.

# HIGH SCHOOL MATHEMATICS COURSES

PUBLIC SCHOOLS OF NORTH CAROLINA DEPARTMENT OF PUBLIC INSTRUCTION

Revised 1998

# High School Grades 9-12

**Number Sense, Numeration, and Numerical Operations**

Students in the secondary years extend understanding of numbers to include real and complex numbers, with attention given to coordinate and vector representations. They develop understanding of more complex representations of numbers, including exponential and logarithmic expressions. They compare and order real numbers, compare rational approximations to exact values, and extend the relationships of primes, factors, multiples, and divisibility to algebraic settings. They express number relationships using positive and negative rational exponents, logarithms, and radicals, and begin to investigate matrices and other advanced applications of numbers.

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**Spatial Sense, Measurement, and Geometry**

Students use appropriate tools, technologies, and constructions to create figures and identify characteristics and properties which describe relationships among shapes. Students use geometric relationships to solve real-world problems such as creating scale drawings and maps. Students also investigate generalizations which include the classic Euclidean properties of triangles and other figures. A variety of proof strategies are used to verify generalizations and properties of shape and connect geometry to other strands of mathematics.

Students further develop the usefulness of measurement from prior years, especially through science and technical course work, while enhancing their facility with the metric system. They focus on solving measurement-related problems involving concepts of precision, tolerance, error, and multiple dimensions.

Students explore intersections of planes and solid figures and examine geometric definitions of conic sections and other standard geometric functions. Students learn basic trigonometric functions for angles and apply trigonometric methods to solve problems involving triangles.

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**Patterns, Relationships, and Functions**

Students continue to use patterns, tables, and graphs to interpret expressions, equations, and inequalities. They use symbols to represent variables, parameters, and functions and extend their use of symbols to include vectors and matrices. Students use technology to assist in developing models and analytical solutions. They use appropriate terminology and notation to define function, domain, range, composition, and inverses of functions. They expand their understanding of functions to include power, quadratic, exponential, periodic, piece-wise, and recursively defined functions. They gather linear and nonlinear data and fit functions to these data using paper-and-pencil methods, graphing calculators, or computers. They interpret the results both in terms of the symbols used and the overall characteristics of the model. They build an understanding of slope as a rate of change. They solve linear and nonlinear equations, inequalities, and systems using algebraic methods as well as intuitive methods, numerical representations, and graphical methods. They simplify algebraic expressions using appropriate methods, including calculators and computers. Students use data, functions, matrices, and other appropriate mathematics to describe, understand, and make predictions about real-world problems.

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**Data, Probability, and Statistics**

Students use appropriate technology such as spreadsheets, statistical software, and graphing calculators, to investigate and analyze data, including comparing data for distinct groups. Students present data in a variety of formats and give a rationale for their choices. They use matrices to represent and manipulate data, and they learn how certain representations can introduce bias into the analysis of data. They use basic measures of central tendency, dispersion, and skewness to describe and analyze data. With bivariate data, they derive and interpret the curve of best fit and use that curve to make predictions. They make hypotheses and test them using arguments based on data and learn to evaluate arguments and conclusions based on data.

Students learn a mathematical formulation for probability which allows them to calculate probabilities of simple and compound events and to determine dependence and independence. They determine probabilities using counting procedures, tables, trees, area models, and formulas for combinations and permutations. Extending their work with discrete mathematics, students use vertex-edge graphs to solve network problems and investigate the connections between networks and matrices. Students use induction, iteration, and recursion to state and solve problems.

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# Introductory Mathematics

**Introductory Mathematics** provides students a survey of preparatory topics for high school mathematics, including the foundations for high school Algebra and Geometry. Appropriate technology, from manipulatives to calculators, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will understand and compute with real numbers and simplify and evaluate algebraic expressions to solve problems.***

1.1 Identify subsets of the real number system.

1.2 Estimate and compute with rational numbers.

1.3 Compare, order, and convert among fractions, decimals (terminating and non-terminating), and percents.

1.4 Solve problems involving percent of increase and percent of decrease.

1.5 Use scientific notation to express large numbers and numbers less than one. Express in standard form numbers given in scientific notation.

1.6 Use rules of exponents.

1.7 Solve problems involving exponents and scientific notation.

1.8 Estimate the square root of a number between two consecutive integers; using a calculator, find the square root of a number to the nearest tenth.

1.9 Determine the absolute value of a number.

1.10 Identify, explain, and apply the commutative, associative, and distributive properties, inverses, and identities in algebraic expressions.

1.11 Simply algebraic expressions.

1.12 Analyze problem situations, select appropriate strategies, and use an organized approach to solve multi-step problems.

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## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use properties and relationships in geometry and standard and customary units of measurement to solve problems.***

2.1 Use geometric concepts and modeling to interpret and solve problems.

2.2 Calculate distances and areas from scale drawings and maps.

2.3 Find the surface area of rectangular solids and cylinders.

2.4 Use models to investigate the relationship of the volume of a cone to a cylinder and a pyramid to a prism with the same base and height.

2.5 Find the volume of prisms, cylinders, pyramids, and cones, with and without models.

2.6 Use the Pythagorean Theorem to solve problems.

2.7 Determine the effect on the volume of solid figures when one or more dimension is changed.

2.8 Solve problems related to similar and congruent figures.

2.9 Locate, give the coordinates of, and graph plane figures which are the results of rotations (multiples of 90°). Graph plane figures which are similar to a given figure (dilations.)

2.10 Identify and draw 3-dimensional figures from different perspectives (top, side, front, corner); use appropriate technology.

2.11 Build 3-dimensional figures given various views.

2.12 Select appropriate units and tools for measurement tasks within problem-solving situations; determine precision and check for reasonableness of results.

## Patterns, Relationships, and Functions

***Goal: The learner will demonstrate an understanding of patterns and simple linear relationships to solve problems.***

3.1 Use formulas in problem-solving situations.

3.2 Solve one and two-step linear equations and inequalities.

3.3 Graph a linear equation by determining ordered pairs. Locate ordered pairs which satisfy a given linear inequality.

3.4 Investigate the concept of slope.

3.5 Describe, extend, analyze and create a wide variety of geometric and numerical patterns, such as Pascal's triangle or the Fibonacci sequence.

## Data, Probability, and Statistics

***Goal: The learner will collect, display, and interpret data to solve problems.***

4.1 Interpret and construct box plots.

4.2 Collect data involving two variables and display on a scatter plot; interpret results; identify positive and negative relationships.

4.3 Interpret the mean, explain its sensitivity to extremes, and explain its use in comparison with the median and the mode.

4.4 Evaluate arguments based on data. Investigate random vs. biased sampling.

4.5 Find the probability of independent and dependent events.

4.6 Make predictions based on theoretical probabilities and experimental results

# Algebra I

**Algebra I** continues the study of algebraic concepts including operations with real numbers and polynomials, relations and functions, creation and application of linear functions and relations, and an introduction to nonlinear functions. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will perform operations with real numbers and polynomials to solve problems.***

1.1 Operate with real numbers to solve a variety of problems.
- a) Apply the laws of exponents to perform operations on expressions with integral exponents.
- b) Evaluate absolute value expressions.
- c) Evaluate radical expressions.
- d) Evaluate algebraic expressions.

1.2 Operate with polynomials.
- a) Add, subtract, and multiply polynomials.
- b) Divide polynomials by monomial divisors.

1.3 Factor polynomials.
- a) Find the greatest common factor of a polynomial.
- b) Factor quadratic expressions.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will solve problems in a geometric context.***

2.1 Use formulas and algebraic expressions (from science, geometry, statistics, etc.) to solve problems.

2.2 Describe, extend, and express algebraically a wide variety of geometric patterns.

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# Patterns, Relationships, and Functions

***Goal: The learner will graph and use relations and functions to solve problems.***

3.1 Translate word phrases and sentences into expressions and equations and vice versa.

3.2 Identify properties and relationships of data in tables, graphs, and equations.

3.3 Define and distinguish between relations and functions, dependent and independent variables, domain and range.

3.4 Graph and interpret in the context of the problem, relations and functions on the coordinate plane. Include linear equations and inequalities, quadratics, and exponentials.

3.5 Determine and use slopes of linear relationships to solve problems.

a) Find the slope of a line given the graph of the line, an equation of the line, or two points on the line.

b) Describe the slope of the line in the context of a problem situation.

3.6 Write the equation of and graph linear relationships given the following information:

a) Slope and y-intercept

b) Slope and one point on the line

c) Two points on the line

3.7 Investigate and determine the effects of changes in slope and intercepts on the graph and equation of a line.

a) Change only slope.

b) Change only the x- or y-intercept.

c) Change the slope and an intercept.

3.8 Use linear equations or inequalities to solve problems. Solve by:

a) Graphing.

b) Using properties of equality; justify steps used.

3.9 Use systems of linear equations or inequalities in two variables to solve problems. Determine the solution by:

a) Graphing.

b) Substitution.

c) Elimination.

3.10 Graph quadratic functions.

a) Locate the intercepts and the vertex.

b) Recognize the x-intercepts of the function as the solutions of the equation.

3.11 Use quadratic equations to solve problems. Solve by:

a) Factoring.

b) Locating points on the graph.

3.12 Use formulas and graphs to solve problems involving exponential functions. Solve a problem by:

a) Locating points on the graph.

b) Evaluating an exponential expression.

## Data, Probability, and Statistics

***Goal: The learner will collect and interpret data to solve problems.***

4.1 Use matrices to display and interpret data.

4.2 Recognize and identify linear and non-linear data.

4.3 Create and use linear models based on real data.

a) Graph the data.

b) Write a linear equation which models a set of real data.

c) Describe the slope and intercepts in the context of the data.

d) Check the model for goodness-of-fit and use the model to make predictions.

# Geometry

**Geometry** continues students' study of geometric concepts building upon middle school topics. Students will move from an inductive approach to deductive methods of proof in their study of geometric figures. Two- and three-dimensional reasoning skills will be emphasized and students will broaden their use of the coordinate plane to include transformations of geometric figures. Appropriate technology, from manipulatives to calculators and graphics software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will perform operations with real numbers to solve problems in a geometric context.***

1.1 Select appropriate operations and solve a variety of application problems using real numbers.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use properties of geometric figures to solve problems and write proofs.***

2.1 Identify, name, and draw sets of points, such as line, ray, segment, and plane.

2.2 Identify the coordinates of a point in a plane or in space.

2.3 Find the length and the midpoint of a segment in two or three dimensions to solve problems.

2.4 Use inductive reasoning and the tools of construction to reach conclusions.

2.5 Use the structure (definitions, postulates, theorems, properties of equality and inequality) of deductive reasoning to solve problems.

2.6 Write and interpret conditional statements including the converse, inverse, and contrapositive.

2.7 Write direct (two-column, paragraph, or flow) and indirect proofs.

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2.8 Use properties, definitions, and theorems of angles and lines to solve problems and write proofs, related to:

a) Adjacent, vertical, linear pair, complementary and supplementary angles.

b) The segment addition postulate and the angle addition postulate.

c) Angle bisectors, segment bisectors, and perpendicular bisectors.

d) Special pairs of angles formed by parallel lines and a transversal.

e) Skew, parallel, and perpendicular lines.

2.9 Use properties, definitions, and theorems of polygons to solve problems related to:

a) Modeling and describing polygons (convex, concave, regular, nonregular).

b) The interior and exterior angles of a convex polygon.

c) Congruent and similar polygons.

2.10 Recognize, identify, and model regular and non-regular polyhedra.

2.11 Use coordinate geometry to confirm properties of polygons.

2.12 Develop and use properties of quadrilaterals (parallelograms, rectangles, rhombi, squares, trapezoids, kites) to solve problems and write proofs.

2.13 Develop and use properties of triangles to solve problems and write proofs related to:

a) The relationships of the lengths of the sides and measures of the angles.

b) Similar triangles and the relationship of their corresponding parts.

c) Congruent triangles and their corresponding parts.

d) Isosceles and equilateral triangles.

e) Altitudes, perpendicular bisectors, angle bisectors, and medians.

2.14 Investigate and use properties of triangles to solve problems and write proofs related to:

a) The interior and exterior angles of a triangle.

b) The segment joining the midpoints of two sides of a triangle.

c) Segments divided proportionally.

2.15 Apply properties of right triangles to solve problems using:

a) The geometric mean.

b) The Pythagorean Theorem and its converse.

c) The relationships in special right triangles.

d) The definitions of sine, cosine, and tangent.

2.16 Develop and use properties of circles to solve problems and write proofs related to:

a) The definition of a circle and sets of points related to the circle.

b) The equation of a circle, its center and radius length.

c) Congruent and concentric circles.

d) Circles and their common tangents.

e) Circumscribed and inscribed figures.

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2.17 Apply properties of circles to solve problems involving:

a) Arcs and angles of circles.

b) The chords, tangents, secants, and radii of a circle.

2.18 Use spheres to solve problems related to the definition of a sphere and sets of points related to the sphere.

2.19 Use formulas to solve problems related to:

a) The perimeter of a geometric figure and circumference of a circle.

b) The area of a triangle, parallelogram, rhombus, trapezoid, square, rectangle, regular polygons, and circles.

c) Arc lengths and the area of sectors of a circle.

d) The ratio of the perimeters, areas, and volumes of similar geometric figures.

e) The lateral area, surface area, and volume of a right prism, pyramid, right circular cylinder, cone, and sphere.

## Patterns, Relationships, and Functions

***Goal: The learner will solve problems with geometric figures in the coordinate plane.***

3.1 Use slopes to determine if two lines are parallel or perpendicular.

3.2 Write the equation of a line parallel or perpendicular to a given line through a given point.

3.3 Transform (translate, reflect, rotate, dilate) polygons in the coordinate plane; describe the transformation in simple algebraic terms.

## Data, Probability, and Statistics

***Goal: The learner will use geometric figures to solve problems involving probability.***

4.1 Use length, area, and volume to solve problems involving probability.

# Algebra II

**Algebra II** continues students' study of advanced algebraic concepts including functions, polynomials, rational expressions, complex numbers, systems of equations and inequalities, and matrices. Emphasis should be placed on practical applications and modeling. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will perform operations with numbers and polynomials to solve problems.***

1.1 Operate with numbers to solve problems.

a) Simplify and perform operations with radical expressions.

b) Simplify expressions involving rational exponents.

c) Use logarithms and exponents to solve problems.

d) Define complex numbers and perform basic operations with them.

1.2 Operate with algebraic expressions to solve problems.

a) Expand powers of binomials using Pascal's triangle or the binomial theorem.

b) Divide one polynomial by another of a lower degree using either synthetic division or the division algorithm.

c) Factor polynomials and other algebraic expressions completely over the real numbers.

d) Find sums, differences, products and quotients of rational algebraic expressions.

e) Simplify complex fractions.

f) Solve problems using direct, inverse, combined and joint variation.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will describe geometric figures algebraically in the coordinate plane.***

2.1 Write the equations in standard form of circles and parabolas; graph.

2.2 Graph ellipses and hyperbolas given the equations.

## Patterns, Relationships, and Functions

***Goal: The learner will use relations and functions to solve problems.***

3.1 Describe graphically, algebraically and verbally real-world phenomena as functions; identify the independent and dependent variables.

3.2 Translate among graphic, algebraic, and verbal representations of relations.

3.3 Graph relations and functions and find the zeros of functions.

3.4 Find the composition and inverse of functions.

3.5 Use quadratic equations and inequalities to solve problems. Solve by:

a) Graphing.

b) Factoring.

c) Completing the square.

d) Using the quadratic formula.

e) Using properties of equality; justify steps needed.

3.6 Find and interpret the maximum and minimum values and the intercepts of a quadratic function.

3.7 Use polynomial equations (up to $4^{th}$ degree) to solve problems. Solve by:

a) Graphing.

b) Factoring;

c) Using properties of equality; justify steps used.

3.8 Find zeros, intercepts, and approximate the turning points of polynomial functions; describe them in the context of the problem.

3.9 Write a polynomial equation given its solutions.

3.10 Use rational equations to solve problems. Solve by:

a) Graphing; identify the asymptotes and intercepts.

b) Factoring.

c) Finding the zeros and asymptotes through analysis of the polynomials in the numerator and denominator.

d) Using properties of equality; justify steps used.

3.11 Use equations which contain radical expressions to solve problems. Solve by:

a) Graphing.

b) Factoring.

c) Using properties of equality; justify steps used.

3.12 Use systems of two or more equations to solve problems. Solve by:

a) Elimination and/or substitution.

b) Graphing.

c) Using matrix equations of the form $AX = B$.

3.13 Use linear programming (systems of three or more inequalities) to solve problems.

3.14 Use equations and inequalities with absolute value to solve problems. Solve by:

a) Locating points on the number line.

b) Locating points on the coordinate plane.

c) Using properties of equality; justify steps used.

3.15 Write and graph exponential functions of the form $f(x) = a\ b^x$.

3.16 Recognize as inverses the exponential and logarithmic functions.

3.17 Use logarithmic and exponential equations to solve problems. Solve by:

a) Graphing.

b) Substitution.

c) Applying the inverse relationship.

d) Using properties of equality; justify steps used.

# Data, Probability, and Statistics

***Goal: The learner will collect, organize, and interpret data with functions of best-fit and matrices to solve problems.***

4.1 Write and interpret an equation of a curve (linear, exponential, quadratic) which models a set of data.

4.2 Find the equation of the curve of best-fit (linear, exponential, quadratic) for a set of data. Interpret the constants, coefficients, and bases in the context of the data. Check the equation for goodness-of-fit and use the equation for predictions.

4.3 Use exponential equations of the form $f(x) = (1+ r)^x$ where r is given as a rate of growth or decay to solve problems.

4.4 Operate with matrices to solve problems.

a) Add, subtract, and multiply matrices.

b) Find the inverse and determinant of a matrix.

# Technical Mathematics 1

**Technical Mathematics 1** continues students' study of algebra, geometry, and probability and statistics, building upon middle school and Algebra I topics. Measurement of two- and three-dimensional figures, special relationships in right triangles, linear and quadratic functions, measures of central tendency, and counting algorithms for probability are the broad topics to be studied in an application-centered environment. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will perform operations with real numbers to solve problems.***

1.1 Solve a variety of application problems using real numbers; use calculators when appropriate.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use properties and relationships in geometry to solve problems.***

2.1 Select and use measuring devices and appropriate units of measurement to solve problems.

2.2 Compare measurements to specified tolerances.

2.3 Use significant digits to indicate accuracy of measurement.

2.4 Interpret and construct maps and scale drawings.

2.5 Solve vector problems using scale drawings. Recognize and indicate magnitude and direction.

2.6 Use formulas to solve problems related to:

a) The perimeter of a geometric figure and circumference of a circle.

b) The area of a triangle, parallelogram, rhombus, trapezoid, square, rectangle, regular polygons, and circle.

c) Arc lengths and the areas of sectors of a circle.

d) The ratio of the perimeters, areas, and volumes of similar geometric figures.

e) The lateral area, surface area, and volume of a right prism, pyramid, right circular cylinder, cone, and sphere.

2.7 Solve for an unknown dimension or an unknown angle in both plane and solid shapes by creating models or diagrams.

2.8 Apply properties of right triangles to solve problems using:

a) Geometric mean.

b) Pythagorean Theorem and its converse.

c) Relationships in special right triangles.

d) Definitions of sine, cosine, and tangent.

## Patterns, Relationships, and Functions

***Goal: The learner will use relations and functions to solve problems.***

3.1 Use slopes to determine if two lines are parallel or perpendicular.

3.2 Write the equation of a line parallel or perpendicular to a given line through a given point.

3.3 Graph relations and functions and find the zeros of functions; use graphing calculators when appropriate.

3.4 Use quadratic equations and inequalities to solve problems. Solve by:

a) Graphing.

b) Factoring.

c) Completing the square.

d) Using the quadratic formula.

e) Using properties of equality; justify steps used.

f) Finding and interpreting the maximum and minimum values and the intercepts of a quadratic function.

3.5 Use linear programming (systems of three or more inequalities) to solve problems.

3.6 Solve problems involving direct and inverse variation.

3.7 Determine domain and range in relations and functions.

## Data, Probability, and Statistics

***Goal: The learner will collect and interpret data to solve problems.***

4.1 Use appropriate measures of central tendency (mean, mode, median) and variance (range, standard deviation) to describe and interpret sets of data.

4.2 Use an appropriate format (table, chart, graph, matrix) to present and analyze data.

4.3 Use formulas for permutations and combinations to determine the number of ways an event can occur.

4.4 Determine the probability of independent and dependent events.

# Technical Mathematics 2

**Technical Mathematics 2** continues students' study of advanced algebraic concepts including linear, quadratic, and exponential functions and matrices. Students will move from an inductive approach to deductive methods of proof in their study of geometric figures. Two- and three-dimensional reasoning skills will be emphasized and students will broaden their use of the coordinate plane to include transformations of geometric figures. Emphasis should be placed on practical applications and modeling. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will perform operations with real numbers to solve problems.***

1.1 Use logarithms and exponents to solve problems.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use properties and relationships in geometry to solve problems.***

2.1 Identify and draw to scale 3-dimensional figures from different perspectives (top, side, front, corner).

2.2 Build 3-dimensional scale models from drawings given various views.

2.3 Identify the coordinates of a point in a plane or in space.

2.4 Find the length and the midpoint of a segment in two or three dimensions.

2.5 Use inductive reasoning and the tools of construction to reach conclusions.

2.6 Use the structure (definitions, postulates, theorems, properties of equality and inequality) of deductive reasoning to solve problems.

2.7 Write direct (two-column, paragraph, or flow) and indirect proofs.

2.8 Use properties and definitions of angles and lines to solve problems related to:

a) Adjacent, vertical, linear pair, complementary and supplementary angles.

b) Angle bisectors, segment bisectors, and perpendicular bisectors.

c) Special pairs of angles formed by parallel lines and a transversal.

d) Skew, parallel, and perpendicular lines.

2.9 Use properties and definitions of polygons to solve problems related to:

a) Modeling and describing polygons (convex, concave, regular, nonregular).

b) The interior and exterior angles of a convex polygon.

c) Congruent and similar polygons.

2.10 Recognize, identify, and model regular polyhedra.

2.11 Use coordinate geometry to confirm properties of polygons.

2.12 Use properties and definitions of quadrilaterals (parallelograms, rectangles, rhombi, squares, trapezoids, kites) to solve problems.

2.13 Use properties and definitions of triangles to solve problems related to:

a) The relationships of the lengths of the sides and measures of the angles.

b) Similar triangles and the relationship of their corresponding parts.

c) Congruent triangles and their corresponding parts.

d) Isosceles and equilateral triangles.

e) Altitudes, perpendicular bisectors, angle bisectors, and medians.

2.14 Investigate and use properties of triangles to solve problems related to:

a) The interior and exterior angles of a triangle.

b) The segment joining the midpoints of two sides of a triangle.

c) Segments divided proportionally.

2.15 Develop and use properties of circles to solve problems involving:

a) The definition of a circle and sets of points related to the circle.

b) The equation of a circle, its center and radius length.

c) Congruent and concentric circles.

d) Circles and their common tangents.

e) Circumscribed and inscribed figures.

2.16 Apply properties of circles to solve problems involving:

a) Arcs and angles of circles.

b) The chords, tangents, secants, and radii of a circle.

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## Patterns, Relationships, and Functions

***Goal: The learner will use relations and functions to solve problems.***

3.1 Transform (translate, reflect, rotate, dilate) polygons in the coordinate plane; describe the transformation in simple algebraic terms.

3.2 Describe graphically, algebraically and verbally real-world phenomena as functions; interpret the independent and dependent variables.

3.3 Solve systems of two or more equations by:

a) Elimination and/or substitution.

b) Graphing.

c) Using matrix equations of the form AX = B.

3.4 Solve problems using combined and joint variation.

3.5 Write and graph exponential functions of the form $f(x) = a\,b^x$.

## Data, Probability, and Statistics

***Goal: The learner will collect and interpret data to solve problems.***

4.1 Use length, area, and volume to solve problems involving probability.

4.2 Write and interpret an equation of a curve (linear, exponential, quadratic) which models a set of data.

4.3 Find the equation of the curve of best-fit (linear, exponential, quadratic) for a set of data. Interpret the constants, coefficients, and bases in the context of the data. Check the equation for goodness-of-fit and use the equation for predictions.

4.4 Use exponential equations of the form $f(x) = (1+r)^x$, where r is given as a rate of growth or decay, to solve problems. Solve by:

a) Locating points on the graph.

b) Evaluating the expression.

4.5 Operate with matrices to solve problems.

a) Use matrices to display and interpret data.

b) Add, subtract, and multiply matrices.

c) Find the inverse and determinant of a matrix.

# Discrete Mathematics

**Discrete Mathematics** introduces students to the mathematics of networks, social choice, and decision making. The course extends students' application of matrix arithmetic and probability. Applications and modeling are central to this course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will solve problems involving social choice and decision making.***

1.1 Solve problems involving election methods: Plurality, Run-off, Sequential Run-off, Borda, Condorcet.

1.2 Solve problems involving weighted voting, voting power, and winning coalitions.

1.3 Solve problems involving estate division.

1.4 Solve problems involving continuous fair division.

1.5 Solve problems involving apportionment: including, but not limited to, Hamilton, Jefferson, Hill, Webster, Adams, Quota (Balinski and Young).

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use graphs to solve problems.***

2.1 Define the terms associated with a graph (edges, vertices, degree, paths, circuits, connected, disconnected, and trees).

2.2 Represent problem situations using finite graphs and adjacency matrices.

2.3 Find the critical path(s) using PERT (Program and Evaluation Review Technique)

2.4 Find an Euler circuit or path, if it exists.

2.5 Find a Hamiltonian circuit or path, if it exists.

2.6 Solve problems involving Euler and Hamiltonian circuits.

2.7 Find the minimum-cost spanning tree for a given graph.

2.8 Incorporate graph coloring to solve real-world problems using the four color theorem and chromatic numbers.

2.9 Use binary expression trees to solve problems in Polish and reverse Polish notation.

2.10 Solve problems involving bin packing.

## Patterns, Relationships, and Functions

***Goal: The learner will use matrices, functions, sequences, and series to solve problems.***

3.1 Solve problems requiring matrix operations.

a) Solve linear systems through applications (Leontief Input-Output Model).

b) Solve problems involving communication networks.

c) Use transition matrices, such as Leslie Matrix and Markov Chains, to make predictions.

d) Use matrices to produce coordinate transformations.

e) Use matrices to determine harvesting strategies to stabilize a population.

3.2 Use recursive relations to solve problems.

3.3 Verify explicit (closed-form) definitions using mathematical induction.

3.4 Find explicit (closed form) definitions using finite differences and geometric or arithmetic formulas.

3.5 Use mixed recursion to solve problems involving growth and decay.

3.6 Use sequences and series to solve problems.

a) Find the sum of a finite sequence.

b) Find the sum of an infinite sequence.

c) Determine if a given series converges or diverges.

d) Represent a series by using sigma notation.

## Data, Probability, and Statistics

*Goal: The learner will solve problems involving counting and probability.*

4.1 Use Venn diagrams to solve counting problems involving intersection and union of sets.

4.2 Use basic laws of logic to solve more complicated Venn diagram problems.

4.3 Solve problems using addition and multiplication principles.

4.4 Solve problems involving permutations and combinations, including independent, dependent, mutually exclusive, and circular.

4.5 Use experimental probability and simulations for probability models.

4.6 Find expected values and determine fairness.

4.7 Identify discrete random variables and use them to solve problems.

4.8 Derive and apply the Binomial Probability Theorem.

# Advanced Mathematics

**Advanced Mathematics** provides students a complete study of trigonometry, as well as advanced algebra topics, analytic geometry, sequences and series, and data analysis. Applications and modeling should be included throughout the course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The student will perform operations with numbers and vectors, and translate between coordinate systems.***

1.1 Convert points in two dimensions between rectangular and polar coordinate systems.

1.2 Operate with vectors in two and three dimensions to solve problems.

a) Add and subtract vectors; multiply vectors by a scaler.

b) Define and find the inner product of vectors.

c) Express vectors as the sum of unit vectors.

1.3 Convert complex numbers between rectangular and polar forms; use DeMoivre's Theorem to find roots and powers of complex numbers.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use trigonometric relationships and transformations to solve problems.***

2.1 Develop and use the trigonometric relationships to solve problems.

a) Determine the values of sine and cosine as represented on the unit circle; include multiples of $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and $\pi$.

b) Find the values of other trigonometric relationships when given the value of one trigonometric relationship.

c) Use the unit circle to develop, recognize, and validate trigonometric identities.

d) Identify the relationship between trigonometry in degree mode and trigonometry in radian mode.

e) Find the radian measure that corresponds to a given angle or arc length.

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2.2 Develop and use the Law of Sines and Law of Cosines to solve problems involving triangles and vectors.

2.3 Use coordinate geometry to describe solid figures.

a) Identify the coordinates of the vertices of polyhedra.

b) Transform polygons in space; describe the results.

c) Transform polygons and polyhedra; use matrix operations to describe the transformation.

## Patterns, Relationships, and Functions

***Goal: The learner will use relations and functions to solve problems.***

3.1 Graph and use the basic functions (constant, linear, quadratic, cubic, square root, absolute value, reciprocal, rational, trigonometric, exponential, logarithmic, piecewise defined, and greatest integer) to solve problems.

a) Compare information given by local behavior versus global behavior.

b) Determine the symmetry of a given graph.

c) Identify continuous and discontinuous functions and locate points of discontinuity.

d) Graph transformations and combinations of transformations for all the functions.

e) Find coordinates of maximum or minimum points of a given function.

f) Write the equation of a function given a set of data or other descriptions of its behavior.

g) Solve equations and inequalities; justify steps used.

h) Compose two functions and find the domain of the composition.

i) Analyze a function by decomposing it into simpler functions.

j) Find the inverse of a function and the domain of the inverse.

3.2 Graph and use the basic quadratic relations (parabola, circle, ellipse, hyperbola) to solve problems.

a) Compare information given by local behavior versus global behavior.

b) Determine the symmetry of a given graph.

c) Identify continuous and discontinuous relations and locate points of discontinuity.

d) Graph transformations and combinations of transformations for all the relations.

e) Find coordinates of maximum or minimum points of a given relation.

f) Write the equation of a relation given a set of data, characteristics, or other descriptions of its behavior.

g) Solve equations and inequalities; justify steps used.

h) Analyze and graph a relation by decomposing it into simpler relations.

i) Find the inverse of a relation and the domain of the inverse.

3.3 Use trigonometric and inverse trigonometric functions to solve problems.

a) Express the tangent, cotangent, secant, and cosecant functions in terms of sine and cosine.

b) Sketch a graph of each of the six trigonometric functions and identify the period of each.

c) Recognize and graph transformations of each of the six trigonometric functions.

d) Use graphs to develop, recognize, and validate trigonometric identities.

e) Solve trigonometric equations and inequalities; justify steps used.

f) Find values of inverse trigonometric functions, applying appropriate domain and range restrictions.

g) Evaluate and graph compositions of trigonometric and inverse trigonometric functions.

3.4 Use polar equations to solve problems.

a) Graph polar equations; identify transformations related to changes in constants and coefficients.

b) Translate quadratic relations between rectangular and polar (parametric) forms; graph.

c) Graph and model real world phenomena using parametric equations.

# Data, Probability, and Statistics

*Goal: The learner will create and use models of data for reporting and analysis.*

4.1 Use sequences and series to solve problems.

a) Find indicated terms in sequences.

b) Use summative notation to describe the sums in a series.

c) Find the sum of a finite series and of an infinite geometric series.

d) Find the limit of an infinite sequence.

e) Find whether a given series converges or diverges.

4.2 Create and use mathematical models of linear, polynomial, exponential, trigonometric, power, and logarithmic functions to solve problems.

a) Linearize data using concepts of composition and inverses in order to find a model for data. Rewrite the linear equation that models linearized data to fit the original curved data.

b) Model growth and decay using recursive relations; compare with $y = ab^x$ and $y = (1 + r)^x$ forms.

c) Use trigonometric functions to model periodic phenomena.

d) Find the model of the curve of best-fit (linear, polynomial, exponential, power, logarithmic, and logistic) for a set of data.

e) Interpret constants, coefficients, and bases in the context of the data being modeled.

f) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

4.3 Summarize distributions of univariate data to solve problems.

a) Determine measures of central tendency (median, mean) and spread (range, standard deviation).

b) Identify data by its position in the distribution (quartiles, percentiles).

c) Recognize, define, and use the normal distribution curve.

# Advanced Placement Statistics

**AP Statistics** introduces students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. Students will observe patterns and departures from patterns, decide what and how to measure, produce models using probability and simulation, and confirm models. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration and Numerical Operations

***Goal: The learner will operate with real numbers to summarize data.***

1.1 Summarize distributions of univariate data by:

a) Measuring center: median and mean.

b) Measuring spread: range, interquartile range, and standard deviation.

c) Measuring position: quartiles, percentiles, and standardized scores (z-scores).

d) Using boxplots.

e) Analyzing the effect of changing units on summary measures.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will display data and use the display for analysis.***

2.1 Interpret graphical displays of distributions of univariate data (dotplots, stemplots, histograms) with attention to:

a) Center and spread.

b) Clusters and gaps.

c) Outliers and other unusual features.

d) Shape.

2.2 Compare distributions of univariate data (dotplots, back-to-back stemplots, parallel boxplots) by comparing:

a) Center and spread: within group, and between group variation.

b) Clusters and gaps.

c) Outliers and other unusual features.

d) Shapes.

## Patterns, Relationships, and Functions

***Goal: The learner will use best-fit functions to study patterns and departures from patterns.***

3.1 Analyze bivariate data using:

a) Scatterplots.

b) Correlation and linearity.

c) Least squares regression line.

d) Residual plots, outliers, and influential points.

e) Transformations to achieve linearity: logarithmic and power transformations.

## Data, Probability, and Statistics

***Goal: The learner will collect data in a well-developed plan and use probability to anticipate the distribution of data.***

4.1 Analyze catagorical data using:

a) Frequency tables.

b) Marginal and joint frequencies for two-way tables.

c) Conditional relative frequencies and association.

4.2 Understand and compare methods of data collection including:

a) Census.

b) Sample survey.

c) Designed experiment.

d) Observational study.

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4.3 Understand principles, methods, and difficulties in sample surveys including:

a) Simple random sampling and systematic sampling.

b) Sampling error (the variation inherent in a survey).

c) Stratifying to reduce variation.

d) Cluster sampling.

e) Sources of bias in a survey.

4.4 Understand principles, methods, and difficulties in designed experiments including:

a) Treatments, experimental units, and randomization;

b) Replication.

c) Control groups.

d) Confounding, placebo effects, blinding.

e) Completely randomized design.

f) Blocked experiments and paired comparison design.

g) Generalizability of results.

4.5 Understand and use probability with:

a) Relative frequency definition of probability;

b) Law of large numbers.

c) Addition rule, multiplication rule, conditional probability, and independence.

d) Discrete random variables and their probability distributions.

e) Simulation of probability distributions, including binomial and geometric.

f) Mean (expected value) and standard deviation of a random variable, including binomial.

g) Mean and standard deviation for sums and differences of independent random variables.

4.6 Understand normal distributions through:

a) Their properties.

b) Models for distributions of measurements.

c) Tables of standard normal probability (Z).

4.7 Understand and simulate sampling distributions for:

a) Sample proportion.

b) Sample mean.

c) Difference between two independent sample proportions.

d) Difference between two independent sample means.

4.8 Understand the Central Limit Theorem and its importance.

4.9 Understand and construct confidence intervals for:

a) Single proportions (using Z).

b) Single means (using Z).

c) Single means (using t distribution).

d) Mean differences from paired samples (using t).

e) Differences between two proportions (using Z).

f) Differences between two independent means (using Z).

g) Differences between two independent means (using t).

h) The slope of the least squares line (using t).

4.10 Understand and perform tests of significance, including:

a) Logic of significance testing, null and alternative hypotheses; p-values; one- and two-sided tests.

b) Test for a proportion (using Z).

c) Test for a mean (using both Z and t).

d) Test for a mean difference from paired samples (using t).

e) Test for a difference between two proportions (using Z).

f) Test for a difference between two independent means (using both Z and t).

g) Test for the slope of the least-squares line (using t).

h) Tests for goodness of fit, homogeneity of proportions, and independence (using chi-square).

# Advanced Placement Calculus

**AP Calculus** develops the student's understanding of the concepts of calculus (functions, graphs, limits, derivatives and integrals) and provides experience with its methods and applications. The course encourages the geometric, numerical, analytical, and verbal expression of concepts, results, and problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration and Numerical Operations

***Goal: The learner will evaluate and operate with expressions and functions.***

1.1 Calculate limits ( including one-sided limits) using algebra.

1.2 Find the slope of a curve at a point.

1.3 Find the derivative of basic functions, including $X^r$, exponential, logarithmic, trigonometric, and inverse trigonometric.

1.4 Use basic rules for derivatives of sum, product and quotient of functions.

1.5 Use chain rule and implicit differentiation.

1.6 Evaluate definite integrals using basic properties.

1.7 Use the Fundamental Theorem to evaluate definite integrals.

1.8 Find the antiderivative of basic functions.

1.9 Find the antiderivative by substitution of variables.

1.10 Use implicit differentiation to find derivatives of inverse functions.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use graphical displays to investigate the geometric behavior of calculus concepts.***

2.1 Use available technology to graph, analyze, predict, and explain observed local and global behavior of functions.

2.2 Recognize asymptotes in terms of graphical behavior.

2.3 Apply geometric understanding of graphs of continuous functions (Intermediate Value Theorem, Extreme Value Theorem).

2.4 Find the equation of tangent lines to a curve at a point and use it to approximate function values.

2.5 Apply Mean Value Theorem and identify its geometric consequences.

2.6 Use derivatives to analyze graphs of curves.

2.7 Evaluate Riemann sums.

2.8 Use the Fundamental Theorem to represent particular antiderivatives both graphically and analytically.

## Patterns, Relationships, and Functions

***Goal: The learner will use use relations, functions, and their derivitives and integrals to solve problems.***

3.1 Describe asymptotic behavior in terms of limits involving infinity.

3.2 Compare relative magnitudes of functions and their rates of change.

3.3 Determine continuity of functions by use of limits.

3.4 Recognize definition of derivative.

3.5 Examine relationship between differentiability and continuity.

3.6 Find instantaneous rate of change as the limit of average rate of change.

3.7 Compare and interpret characteristics of graphs of f, f', and f''.

3.8 Model a written description of a physical situation with a differential equation and vice-versa.

3.9 Use derivatives to solve optimization problems.

3.10 Use derivatives to model rates of change.

3.11 Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.

3.12 Interpret the definite integral as a limit of Riemann sums.

3.13 Recognize the definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

3.14 Use integrals to model physical, social, or economic situations. Specific applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, and the distance traveled by a particle along a line.

3.15 Find a specific antiderivative using initial conditions.

3.16 Solve separable differential equations and use them as models.

## Data, Probability, and Statistics

***Goal: The learner will use graphs and tables of data to investigate concepts of calculus.***

4.1 Estimate limits from graphs or tables of data.

4.2 Approximate rates of change from graphs and tables of values.

4.3 Approximate definite integrals numerically with Riemann sums and Trapezoidal Rule.

# Integrated Mathematics 1

**Integrated Mathematics 1** provides students the opportunity to study traditional topics from algebra, geometry, probability, and statistics in a problem-centered, connected approach. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will perform operations with real numbers to solve problems.***

1.1 Apply the laws of exponents to perform operations on expressions with integral exponents.

1.2 Evaluate algebraic expressions including absolute value and radical expressions.

1.3 Solve problems using direct and inverse variation.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use properties of geometric figures to solve problems and write proofs.***

2.1 Identify, name, and draw sets of points, such as line, ray, segment, and plane.

2.2 Identify the coordinates of a point in a plane or in space.

2.3 Use inductive reasoning and the tools of construction to reach conclusions.

2.4 Write and interpret conditional statements including the converse, inverse, and contrapositive.

2.5 Use properties and definitions of polygons to solve problems related to:

a) Modeling and describing polygons (convex, concave, regular, nonregular).

b) The interior and exterior angles of a convex polygon.

c) Congruent and similar polygons.

2.6 Investigate and use properties of triangles to solve problems and write proofs related to:

a) The interior and exterior angles of a triangle.

b) The segment joining the midpoints of two sides of a triangle.

c) Segments divided proportionally.

2.7 Develop and use properties of triangles to solve problems and write proofs related to:

a) The relationships of the lengths of the sides and measures of the angles.

b) Similar triangles and the relationship of their corresponding parts.

c) Congruent triangles and their corresponding parts.

d) Isosceles and equilateral triangles.

e) Altitudes, perpendicular bisectors, angle bisectors, and medians.

2.8 Use formulas to solve problems related to:

a) The perimeter of a geometric figure and circumference of a circle.

b) The area of a triangle, parallelogram, rhombus, trapezoid, square, rectangle, regular polygons, and circles.

c) Arc length and the area of the sector of a circle.

d) The ratio of the perimeters, areas, and volumes of similar geometric figures.

e) The lateral area, surface area, and volume of a right prism, pyramid, right circular cylinder, cone, and sphere.

## Patterns, Relationships, and Functions

***Goal: The learner will graph and use relations and functions to solve problems.***

3.1 Translate word phrases and sentences into expressions and equations and vice versa.

3.2 Identify properties and relationships of data in tables, graphs, and equations.

3.3 Define and distinguish between relations and functions, dependent and independent variables, domain and range.

3.4 Graph and interpret in the context of the problem, relations and functions on the coordinate plane. Include linear equations and inequalities, and exponentials.

3.5 Determine and use slopes of linear relationships to solve problems.

a) Find the slope of a line given the graph of the line, an equation of the line, or two points on the line.

b) Describe the slope of the line in the context of a problem situation.

3.6 Write the equation of and graph linear relationships given relevant information.

a) Slope and y-intercept.

b) Slope and one point on the line.

c) Two points on the line.

3.7 Investigate and determine the effects of changes in slope and intercepts on the graph and equation of a line.

a) Change only slope.

b) Change only the x- or y-intercept.

c) Change the slope and an intercept.

3.8 Use linear equations or inequalities to solve problems. Solve by:

a) Graphing.

b) Using properties of equality; justify steps used.

3.9 Use systems of linear equations or inequalities in two variables to solve problems. Determine the solution by:

a) Graphing.

b) Substitution.

c) Elimination.

3.10 Use formulas and graphs to solve problems involving exponential functions. Solve a problem by:

a) Locating points on the graph.

b) Evaluating an exponential expression.

## Data, Probability, and Statistics

***Goal: The learner will use data summaries, probability, and linear models to investigate data and solve problems.***

4.1 Use basic measures of central tendency (median, mean) and dispersion (range, standard deviation) to describe and analyze data.

4.2 Use length, area, and volume to solve problems involving probability.

4.3 Recognize and identify linear and non-linear data.

4.4 Create and use linear models based on real data.

a) Graph the data.

b) Write a linear equation which models a set of real data.

c) Describe the slope and intercepts in the context of the data.

d) Check the model for goodness-of-fit and use the model to make predictions.

# Integrated Mathematics 2

**Integrated Mathematics 2** continues students' study of traditional topics from algebra, geometry, probability, and statistics in a problem-centered, connected approach. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will perform operations with numbers and polynomials to solve problems.***

1.1 Operate with numbers to solve problems.
- a) Simplify and perform operations with radical expressions.
- b) Simplify expressions involving rational exponents.
- c) Define complex numbers and perform basic operations with them.

1.2 Operate with algebraic expressions to solve problems.
- a) Add, subtract, and multiply polynomials.
- b) Divide polynomials by monomial divisors.
- c) Expand powers of binomials using Pascal's triangle or the binomial theorem.
- d) Divide one polynomial by another of a lower degree using either synthetic division or the division algorithm.
- e) Factor polynomials and other algebraic expressions completely over the real numbers.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use properties of geometric figures to solve problems and write proofs.***

2.1 Find the length and the midpoint of a segment in two or three dimensions to solve problems.

2.2 Use the structure (definitions, postulates, theorems, properties of equality and inequality) of deductive reasoning to solve problems.

2.3 Write direct (two-column, paragraph, or flow) and indirect proofs.

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2.4 Use coordinate geometry to confirm properties of polygons.

2.5 Transform (translate, reflect, rotate, dilate) polygons in the coordinate plane; describe the transformation in simple algebraic terms.

2.6 Use properties, definitions, and theorems related to angles and lines to solve problems and write proofs, regarding:

a) Adjacent, vertical, linear pair, complementary and supplementary angles.

b) The segment addition postulate and the angle addition postulate.

c) Angle bisectors, segment bisectors, and perpendicular bisectors.

d) Special pairs of angles formed by parallel lines and a transversal.

e) Skew, parallel, and perpendicular lines.

2.7 Use properties, definitions, and theorems related to quadrilaterals (parallelograms, rectangles, rhombi, squares, trapezoids, kites) to solve problems and write proofs.

2.8 Apply properties of right triangles to solve problems using:

a) Geometric mean.

b) Pythagorean Theorem and its converse.

c) Relationships in special right triangles.

d) Definitions of sine, cosine, and tangent.

## Patterns, Relationships, and Functions

***Goal: The learner will graph and use relations and functions to solve problems.***

3.1 Describe graphically, algebraically and verbally real-world phenomena as functions; identify the independent and dependent variables.

3.2 Graph relations and functions and find the zeros of functions.

3.3 Use quadratic equations to solve problems. Solve by:

a) Graphing.

b) Factoring.

c) Completing the square.

d) Using the quadratic formula.

e) Using properties of equality; justify steps used.

3.4 Find and interpret the maximum and minimum values and the intercepts of a quadratic function; recognize the x-intercepts of the function as the solutions of the equation.

3.5 Use slopes to determine if two lines are parallel or perpendicular.

3.6 Write the equation of a line parallel or perpendicular to a given line through a given point.

3.7 Use systems of two or more equations to solve problems. Solve by:

a) Elimination, substitution.

b) Graphing.

c) Using matrix equations of the form AX = B.

3.8 Use linear programming (systems of three or more inequalities) to solve problems.

3.9 Write and graph exponential functions of the form $f(x) = a\ b^x$.

## Data, Probability, and Statistics

***Goal: The learner will use probability and matrices to solve problems.***

4.1 Define and use permutations and combinations in counting situations.

4.2 Perform experiments and compare theoretical and empirical results.

4.3 Calculate the expected value of an event; use expected value to solve problems.

4.4 Operate with matrices to solve problems.

a) Use matrices to display and interpret data.

b) Add, subtract, and multiply matrices; use graphing calculators when appropriate.

c) Find the inverse and determinant of a matrix; use graphing calculators when appropriate.

# Integrated Mathematics 3

**Integrated Mathematics 3** provides students a problem-centered, connected study of advanced algebra topics, geometry, and data analysis. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Number Sense, Numeration, and Numerical Operations

***Goal: The learner will operate with numbers and algebraic expressions to solve problems.***

1.1 Use logarithms and exponents to solve problems.

1.2 Operate with algebraic expressions to solve problems.

a) Find sums, differences, products and quotients of rational algebraic expressions.

b) Simplify complex fractions.

c) Solve problems using combined and joint variation.

## Spatial Sense, Measurement, and Geometry

***Goal: The learner will use properties of geometric figures to solve problems and write proofs.***

2.1 Use properties, definitions, and theorems related to circles to solve problems and write proofs involving:

a) The definition of a circle and sets of points related to the circle.

b) The equation of a circle, its center and radius length.

c) Congruent and concentric circles.

d) Circles and their common tangents.

e) Circumscribed and inscribed figures.

2.2 Apply properties of circles to solve problems involving:

a) Arcs and angles of circles.

b) Chords, tangents, secants, and radii of a circle.

2.3 Use spheres to solve problems related to the definition of a sphere and sets of points related to the sphere.

2.4 Recognize, identify, and model regular and non-regular polyhedra.

2.5 Use coordinate geometry to describe solid figures.

a) Identify the coordinates of the vertices of polyhedra.

b) Transform polygons in space; describe the results.

c) Transform polyhedra in space; describe the transformation in algebraic terms.

2.6 Write the equations in standard form of circles and parabolas and graph.

2.7 Graph ellipses and hyperbolas given the equations.

## Patterns, Relationships, and Functions

***Goal: The learner will graph and use relations and functions to solve problems.***

3.1 Translate among graphic, algebraic, and verbal representations of relations.

3.2 Find the composition and inverse of functions.

3.3 Use equations, and inequalities with absolute value to solve problems by:

a) Locating points on the number line.

b) Locating points on the coordinate plane.

c) Using properties of equality and justifying steps needed.

3.4 Use polynomial equations (up to $4^{th}$ degree) to solve problems. Solve by:

a) Graphing.

b) Factoring.

c) Finding zeros, intercepts, and approximate the turning points of polynomial functions; describe in the context of the problem.

d) Using properties of equality; justify steps used.

e) Write a polynomial equation given its solutions.

3.5 Solve problems involving rational equations. Solve by:

a) Graphing; identify the asymptotes and intercepts.

b) Factoring.

c) Finding the zeros and asymptotes through analysis of the polynomials in the numerator and denominator.

d) Using properties of equality; justify steps used.

3.6 Solve equations which contain radical expressions. Solve by:

a) Graphing.

b) Factoring.

c) Using properties of equality; justify steps used.

3.7 Recognize as inverses the exponential and logarithmic functions.

3.8 Use logarithmic and exponential equations to solve problems. Solve by:

a) Graphing.

b) Substitution.

c) Applying the inverse relationship.

d) Using properties of equality; justify steps used.

## Data, Probability, and Statistics

***Goal: The learner will collect, organize, and interpret data with functions of best-fit to solve problems.***

4.1 Write and interpret an equation of a curve (linear, exponential, quadratic) which models a set of data.

4.2 Find the equation of the curve of best-fit (linear, exponential, quadratic) for a set of data. Interpret the constants, coefficients, and bases in the context of the data. Check the equation for goodness-of-fit and use the equation for predictions.

4.3 Use exponential equations of the form $f(x) = (1+r)^x$ where $r$ is given as a rate of growth or decay to solve problems.